Session 6

Magnetic fields

Dr Sarah Symons

Physics Innovations Centre for Excellence in Learning and Teaching

πCETL (Leicester)

Department of Physics and Astronomy

University of Leicester

Contents

Welcome

In the last session we looked at electric fields. This is the first of two sessions in which we shall study magnetic fields. In session 8 we'll put the electric and magnetic fields together to get Maxwell's electromagnetic theory.

In this session we'll begin with Faraday's law of electromagnetic induction. We'll describe and use the analogy between a current loop and a magnetic dipole and study the magnetic energy in various situations, including the energy density of a magnetic field. Finally we'll look at magnetic forces from the point of view of the interaction between moving electrical charges, the Lorentz force law, and describe the force between current carry wires, the Biot-Savart law.

Session Author

Dr Sarah Symons, University of Leicester.

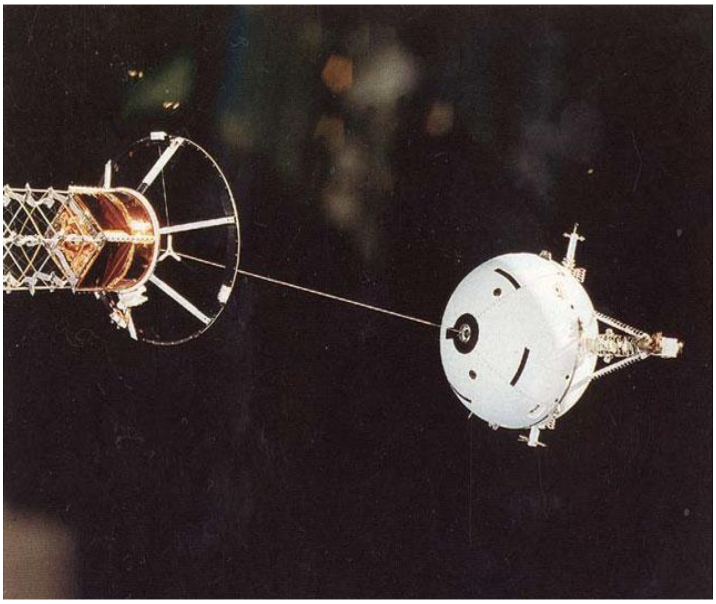
Session Editor – Tim Puchtler

Learning Objectives

- Describe and apply Faraday’s law of electromagnetic induction
- Describe and use the analogy between a current loop and a dipole
- Use the formulae for energy density of the magnetic field
- Describe magnetic forces in terms of the interaction of moving charges (Lorentz force)
- Describe the force between current carrying wires (Biot-Savart Law)

The Problem

For the problem in this session we return to space. The statement of our problem is illustrated in the pictures. What do these pictures illustrate? Let's toss around some ideas:




Image[1]

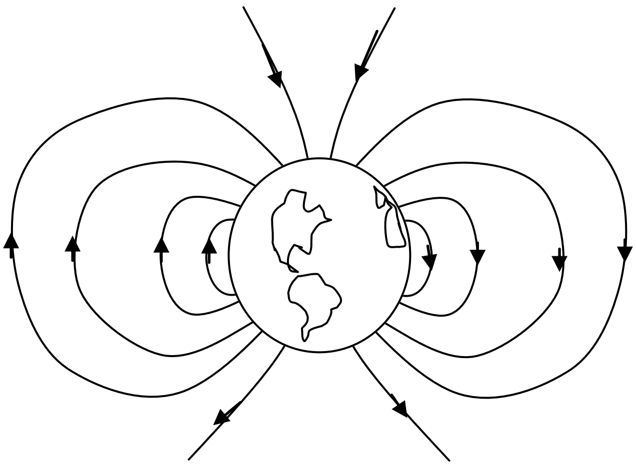
Is this a refueling operation? It could be, but why use such a long connector? Is this a power source then? A way of boosting payloads into space? A braking mechanism? Actually it could be all of these. But then in all cases it must be extracting energy from the environment. So let's ask a different question: what is the tether interacting with? If the tether is a conductor the most likely candidate is the Earth's magnetic field. So we'll start with this.

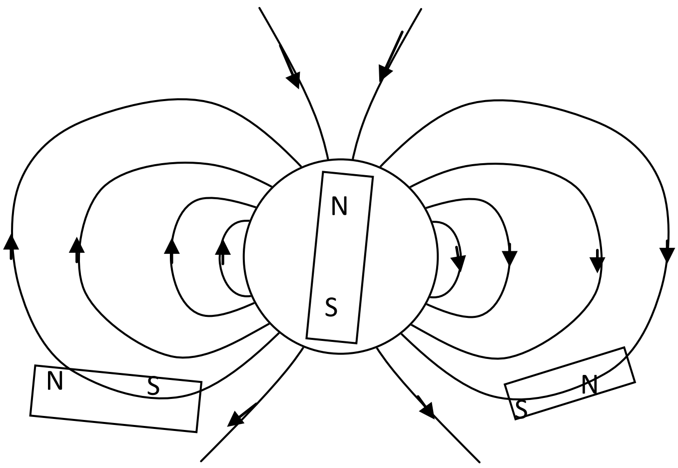
[1] Images credited to TSS-1, STS-46 Crew, NASA, http://www.nasa.gov/vision/universe/roboticexplorers/tethered_spacecraft.html

Dipole Fields

The Earth's Magnetic Fields

The presence of a magnetic field at the surface of the Earth has been known since ancient times, although first studied in the 17th Century. The field is not uniform over the surface – it changes in strength by a factor of about two and also varies in angle. By studying these variations we can deduce that the field approximates that of a bar magnet. Although the field is not exactly that of a bar magnet, we can use this approximation to deduce how the field falls off in strength as we go away from the surface of the Earth into space.



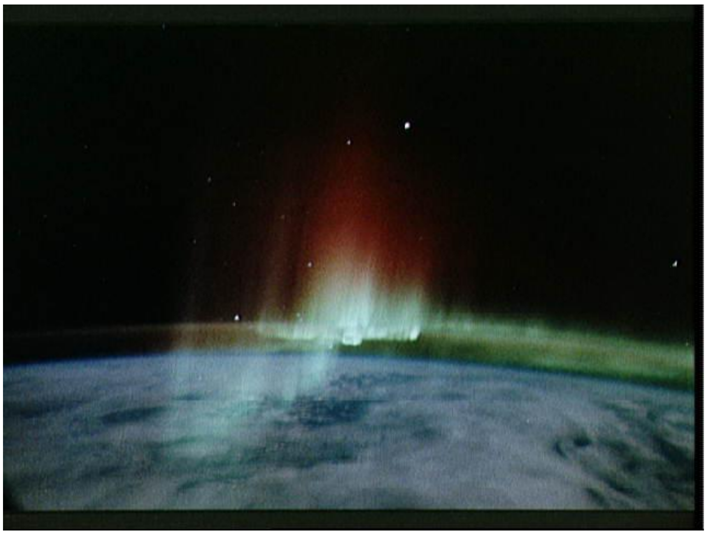


The figures give an approximate representation of the Earth's field as similar to that of a bar magnet. We'll have to think about what the lines and arrows mean. For an electric field they represented the force on a unit charge. For magnetism the analogue of a unit electrical charge is not a unit magnetic charge – magnetic charge does not exist. For the moment we can rely on our intuitive picture of the field lines as pointing in the direction of a compass needle.

The field lines in the pictures represent an approximately dipole field. As we'll see shortly, an exact electrical dipole is obtained by placing a positive and a negative charge close together, so that field lines appear to start and end at adjacent points. A magnetic dipole is similar in that the field lines start and end at adjacent points, which we call the south and north poles. The Earth's field is not produced by a bar magnet, but we'll see later why it appears as though it is, at least to a first approximation.

How do we know the field is there?

As well as the compass we get some picturesque evidence of the Earth's field from the aurora. These result from charged particles in the Earth's magnetosphere spiraling down magnetic field lines and colliding with oxygen and nitrogen atoms in the Earth's atmosphere. These collisions excite the gas atoms, which then de-excite by emitting visible radiation – green and red in the case of atomic oxygen. The aurora are centred on the magnetic poles not the geographical ones, confirming the association with the Earth's magnetic field.

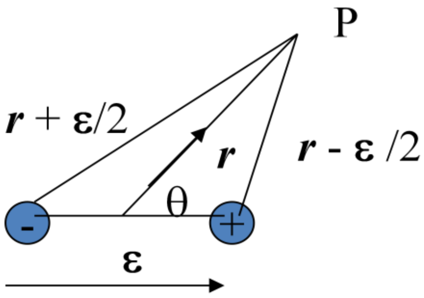


Image[2]

Dipole Fields

To determine the magnetic field in space we need to know how it falls off with altitude. Contrary to what many students appear to think, not all fields fall off as the inverse square of the distance. The Earth's field is at least approximately a dipole. So our first task is to study dipoles. We'll illustrate this with an electric dipole. This consists of a positive and negative charge, $+q$ and $-q$, in close proximity. In the figure they are a distance ε apart.

[2] Dueling Auroras, Credited to NASA



The dipole moment of the charge distribution is the moment of the charges about their centre, which is here twice q time ε/2 or q ε. The potential of the charges at the field point P is given by the sum of the Coulomb potentials for each charge, which is expressed in equation (1). This can be approximated by using the binomial theorem as shown in equation (2), provided that the field point is sufficiently far away from the dipole so that r >> ε.

$$4\pi\varepsilon_0 V = \frac{q}{\left(r^2 + r\varepsilon\cos\theta + (\varepsilon/2)^2\right)^{1/2}} - \frac{q}{\left(r^2 - r\varepsilon\cos\theta + (\varepsilon/2)^2\right)^{1/2}} \quad (1)$$

$$\approx \frac{q}{r}\left[\left(1 - \frac{\varepsilon r\cos\theta}{2r^2}\right) - \left(1 + \frac{\varepsilon r\cos\theta}{2r^2}\right)\right] \quad (2)$$

$$= \frac{q\varepsilon\cos\theta}{r^2} = \frac{\boldsymbol{d}.\hat{\boldsymbol{r}}}{r^2}$$

where $\boldsymbol{d} = q\varepsilon$ is the dipole moment

and $\hat{\boldsymbol{r}}$ Is a unit vector in the direction of $\boldsymbol{r}$

The field is found by differentiating the potential. The radial component is just the derivative with respect to r. The component in the θ direction – the azimuthal component – is a bit more complicated – it's the change in V per unit ***distance*** rdθ in the θ direction, not per unit angle. Hence it's 1/r times the derivative of the potential with respect to θ, not just dV/dθ.

Thus the radial component of the field is $-\frac{dV}{dr} = \frac{2\boldsymbol{d}.\hat{\boldsymbol{r}}}{4\pi\varepsilon_0 r^3} = \frac{2d\cos\theta}{4\pi\varepsilon_0 r^3}$

The azimuthal component is $-\frac{1}{r}\frac{dV}{d\theta} = \frac{d\sin\theta}{4\pi\varepsilon_0 r^3}$

A dipole field falls of as 1/r^3

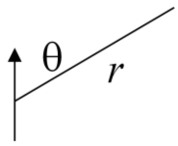
Magnetic Dipoles

In the case of magnetic fields we do not have the analogue of charges – magnetic poles do not exist. So we use the field structure we have just derived as the **definition** of the field of a magnetic dipole. Thus, we derive the magnetic field of a magnetic dipole of strength m from the derivative of $\mu_0 m\cos\theta/r^2$.

You need to be careful with units here: the product of current I times area A has SI units of amps m^2. Sometimes you'll see the dipole moment quoted as Tesla m^3. This is not the same – the latter units arise when the magnetic moment is taken to be what we have called $\mu_0 m$ – in this case the formula would not have the factor μ_0 explicitly.

However, all this is of little use if we do not have some way of finding the dipole moment m. So we'll turn to that next.

$$B_r = -\frac{\partial}{\partial r}\left(\frac{\mu_0 m\cos\theta}{4\pi r^2}\right), \quad B_\theta = -\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{\mu_0 m\cos\theta}{4\pi r^2}\right)$$



where m is the magnetic dipole moment

SI unit of: **B is Tesla (T)**

m is A m^2 (amp metre^2)

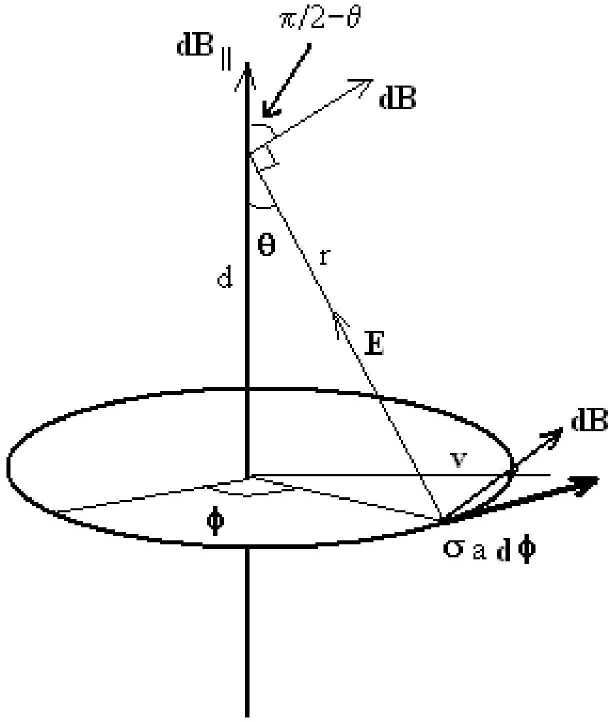
μ_0m is T m^3

$\mu_0 = 4\pi \times 10^{-7}$ N A^{-2}

What is the field on the axis of a dipole at distance *r* ?

Current Loops and Magnetic Dipoles

Recall that magnetic fields originate from moving charges. Thus, we can ask the question, what configurations of moving charges give rise to a dipole field? The answer turns out to be current loops. We can find the magnetic field of a small loop of current from the Biot-Savart law of session 5. By comparison with the dipole field we can then deduce the expression for the dipole moment of the loop. From one point of view therefore, current loops play a fundamental role in magnetism similar to that played by elementary charges in electrostatics.



To derive this result we start by dividing the current loop of radius *a*, carrying a charge σ per unit length, into small elements of charge σadϕ. We assume these elementary charges are moving with speed v in order to create the current I in the loop. The field at the point P due to the small element adϕ is given by the Biot-Savart law for the field of a moving charge, equation (1). For off-axis field points the calculation is difficult, so we'll restrict attention to the field on the axis of the loop as shown in the figure.

$$dB = \frac{\mathbf{v} \wedge d\mathbf{E}}{c^2} \text{..........................(1)}$$

$$dB_{\parallel} = \frac{v(\sigma a d\phi)\sin\theta}{4\pi\varepsilon_o r^2 c^2} \text{....................(2)}$$

$$= \frac{\mu_o v \sigma a d\phi \sin\theta}{4\pi\varepsilon_o r^2}; \quad \sin\theta = a/r\ ; c^2 = \frac{1}{\varepsilon_o \mu_o}$$

$$B_{\parallel} = \frac{2\mu_o I \pi a^2}{4\pi r^3} \text{.........................(3)}$$

$$= \frac{2\mu_o IA}{4\pi r^3} \text{.............................(4)}$$

The working for these equations is explained below.

We begin by integrating equation (1) round the loop to sum up the effect of all the elementary charges. It should be clear from the figure that the component of B perpendicular to the axis sums to zero since the contribution from one side of the loop cancels a similar contribution from the opposite side. We therefore calculate just the component of B **along** the axis, which we've called $B_{//}$. This brings in the factor $\cos(\pi/2 - \theta)$ or $\sin\theta$. For the electric field E, we substitute the field of a point charge of magnitude $\sigma a d\phi$. This gives us equation (2).

Now we have to add the fact that ε_0, μ_0, and c^2 are related by $c^2 = 1/\varepsilon_0\mu_0$. We'll see how this comes about in a future session, but for the moment we'll just take it as a fact – a numerical fact if you like. We also have an expression for $\sin\theta$ as a/r. Finally we need the fact that σv is the charge passing a point every second, so it's just the current in the loop, I. Putting this together gives us (3). Identifying πa^2 as the area of the loop A gives us (4).

We can compare this with the electric field of a dipole. The two expressions are the same, excluding the constants ε_0 and μ_0 which just look after the units, if the dipole moment is replaced by IA.

$$E_d = \frac{2d}{4\pi\varepsilon_o r^2} \Rightarrow m = IA$$

Off axis the field of the loop is only a dipole if we are far enough away that the loop can be treated as almost a point.

A current loop is a magnetic dipole of moment *IA*

Energy of the Field

Since we are going to talk about extracting energy from the magnetic field, it will be useful to identify the energy of the field.

As we've said many times, the fundamental approach to physics involves postulating an energy for each of the fundamental fields. (This isn't as difficult as it sounds – there is only one standard pattern that works, modulo a few thematic variations.) For the magnetic field the energy is $B^2/2\mu_0$ per unit volume.

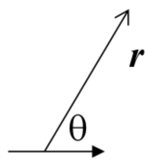
Energy per unit volume has the same dimensions as force per unit area. So this must also be the order of magnitude of the pressure exerted by a magnetic field on moving charges. It's not exactly the pressure, because dimensional considerations don't tell us about any numerical constant (which is actually 1/3 here).

The energy per unit volume of the magnetic field is $\frac{B^2}{2\mu_0}$

Energy per unit volume = force per unit area, so this is also the order of magnitude of the pressure exerted by a magnetic field

Summary

The potential of an electric dipole is $\frac{\mathbf{d}.\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2} = \frac{d\cos\theta}{4\pi\varepsilon_0 r^2}$



The potential of a magnetic dipole is $\frac{\mu_0 \mathbf{m}.\hat{\mathbf{r}}}{4\pi r^2} = \frac{\mu_0 m\cos\theta}{4\pi r^2}$

A dipole field falls of as $1/r^3$

The energy of a magnetic field is $\frac{B^2}{2\mu_0}$ per unit volume

SAQs

1. A typical magnetic field at the surface of the Earth is $0.5\ 10^{-4}$ T. What is the magnetic moment, m? Give your answer in Am^2 to the nearest integer, times a power of 10, in the form n, m
2. Estimate the current in amps in the Earth required to produce the Earth's magnetic field to the nearest power of 10. (Enter your answer as the integer power.) Note: The Magnetic moment of Earth, $\mu_0 m = 5\ 10^{16}\ Tm^3$
3. The solar wind exerts a pressure of 1.7×10^{-9} Pa. At about how many Earth radii is this balanced by the pressure of the Earth's magnetic field? Give your answer to the nearest integer.

The answers appear on the following page

Answers

1. 7, 22 (6 to 8 , 22 is acceptable) Use the formula for a dipole field at a radius of 6378 km.

2. Substitute for m from question 1 in the formula m = IA. The answer may depend on what you take for the core radius, R_c. The liquid core extends to 3400 km which is the value we've used with $A = \pi R_c^2$.

3. 10 (8 to 12 is acceptable). The pressure is of order $B^2/2\mu_0$. Calculate the value at the surface and multiply by $(R_E/r)^6$ (where R_E is the radius of the Earth) to get the pressure at r where this balances the solar wind. Note the question asks for the number of Earth radii r/R_E.

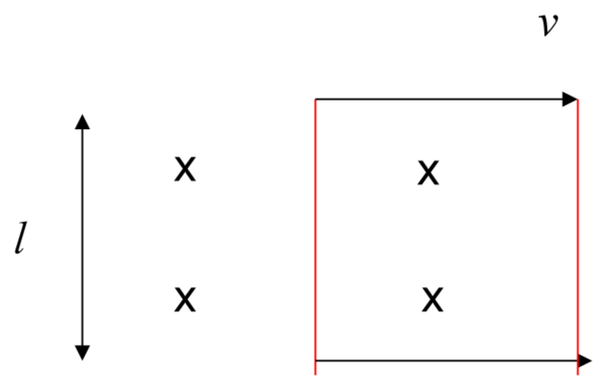
Magnetic Induction

Faraday's Law

Here we come to one of the fundamental laws linking electric and magnetic fields, Faraday's law of induction.

Before we start, recall the meaning of the flux of a field through a given area – the flux of a field is the strength of the field times the area.

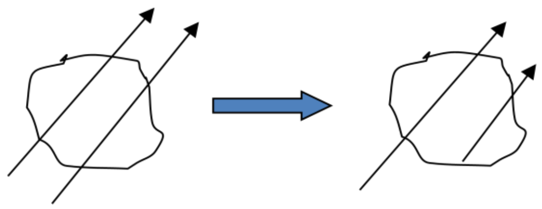
There are two pictures which give the same result:



In the first we take a length of conductor and move it across a fixed magnetic field. The directions are important – only the field perpendicular to the plane of the motion matters. In the image, the conductor moves across the page and the field points into it. This cutting of field lines induces a potential difference between the ends of the conductor. Faraday's law says that the induced potential difference equals the rate of change of the flux of the magnetic field.

In 1 second the moving wire links flux $vBl = d/dt(BA) = \dot{\Phi} = V$

In the second picture we take a fixed loop of conductor and place it in a time varying magnetic field. This time the rate of change of flux comes from the changing field. Not the movement of the conductor, but the result is the same: Faraday's law says that the induced potential difference equals the rate of change of the flux of the magnetic field.



$$\dot{\Phi} = A\,\mathrm{d}B/\mathrm{d}t = V$$

Feynman describes it as a remarkable coincidence that these two different situations should follow exactly the same law.

We are now in a position to address the problem of the tether. This is a conductor moving through the Earth's magnetic field. Faraday's law will give us the induced potential difference. Try making your own estimate of the potential difference from the data before you look at mine.

There are several ways of approaching this. I prefer to get a formula for the result before substituting numbers: it's easier to correct mistakes that way. I've also avoided having to look up data for big G and the mass of the Earth. I've calculated the speed of the spacecraft from what we learnt in session 2; but, of course, I could also have guessed a value for the speed of a spacecraft on the grounds that they orbit in about 90 minutes at the Shuttle altitude.

Data:

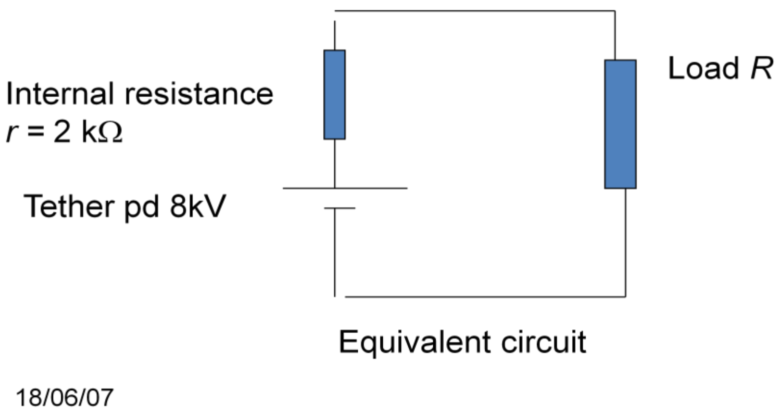
Earth's field at surface: $B_E = 0.5\ 10^{-4}$ T
Field at low Earth orbit, say 200km : $B_E(R_E/r)^3$ – differs from B_E by about 10% so take as B_E
Speed of spacecraft $v = (GM/r)^{1/2} = (GM/R_E^2 \times R_E^2/r)^{1/2} \sim (gR_E)^{1/2}$
Length of tether = l = 20 km
EMF = $Bvl = B_E(gR_E)^{1/2}l$ = 8 kV

Whatever the value you got, it should be quite large. But what use is a potential difference?

Recall that the air at this height is ionised by solar radiation – so it is a plasma, that is an electrically conducting gas. This provides a return path for current. In other words, we can complete the circuit and extract power from the tether.

Extracting Power

Suppose we try to extract power from the tether by connecting to some load. Given that the tether acts as a battery with internal resistance, what is the maximum power available? We looked at this in one of the additional problems of session 4. The maximum power occurs when the load impedance is matched to the impedance of the battery. For a 200km tether the internal resistance is 2000 ohms, so total resistance will be 4 kilohms; 8 kilo Volts then supplies a current of 2 amps. This gives a power, I^2R, of 8kW in the external load and 8kW dissipated in the tether.



Are we getting something for nothing here? Where is the energy coming from?

The cutting of magnetic field lines by the tether induces a potential difference. This gives rise to a flow of current in the tether. The action of the magnetic field on this current will now produce a force on the tether. We'll look at this in the next section and then consider the conservation of energy.

Summary

Faraday's law: the potential difference induced in a circuit equals the rate of change of magnetic flux linked to the circuit

$$V = \frac{d\Phi}{dt}$$

SAQs

1. A plane circular coil of radius a is threaded by a uniform magnetic field $B = B_0 \sin \omega t$ normal to the plane of the coil. What is the pd induced in the coil?
   a) $2\pi aB$
   b) $\pi a^2 B$
   c) 0
   d) $\frac{1}{2}\pi a^2 \omega B_0$
   e) $\pi a^2 \omega B_0 \sin \omega t$

2. A circular coil of radius a spins at n revs per second about a horizontal axis in a uniform vertical magnetic field B. What is the pd induced in the coil?
   a) 0
   b) $\pi a^2 n B$
   c) $2\pi^2 a^2 n B \sin 2\pi n t$

The answers appear on the following page

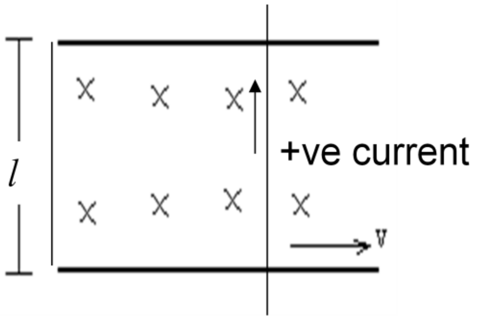
Answers

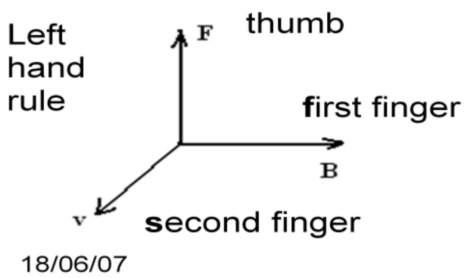
1. a) Wrong: You're thinking of the circulation – the induced voltage involves the flux of the field.
   b) Wrong: this is the flux of B, not the rate of change of flux which is what you want
   c) Wrong: there is a pd round the circuit, which will drive a current
   d) Wrong: you're thinking of the rms value, which is not what is asked for
   e) Correct: this is the time derivative of the flux

2. a) Wrong: the fact that there is a closed circuit does not mean there is no pd round it.
   b) Wrong: this is the flux times n which is not the same as the rate of change of flux
   c) Correct: the normal component of the field must be $B\cos 2\pi n t$ so the time derivative of the flux is as given. If you got $2\pi^2 a^2 n B \cos 2\pi n t$ this is also right if you're measuring time from the moment the coil and field are parallel rather than perpendicular.

Magnetic Forces

We start from Faraday's law for the potential difference induced in a conductor by a changing, magnetic flux. Let's once again imagine a conductor forming part of a circuit, moving through a magnetic field B with speed v. The **magnitude** of the potential difference induced in the circuit is d(phi)/dt. If a charge q moves down this potential the work done is qV. The nub of the calculation is to put the work done equal to the force times distance, and thereby find the force on the moving charge.





$$V = \frac{d\Phi}{dt} \quad \text{and} \quad \Phi = AB$$

$$\text{Work done } qV = \frac{d(q\Phi)}{dt} = qB\frac{dA}{dt}$$

$$\text{But Work} = \text{Force} \times \text{distance, so } Fl = qB\frac{dA}{dt} \ldots\ldots\ldots\ldots (1)$$

$$= qBlv$$

$$\text{or} \qquad F = qvB \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Direction: to induce the flow of charge in the wire

In vectors: $\mathbf{F} = q\mathbf{v} \wedge \mathbf{B}$ - Lorentz force on a charge, q

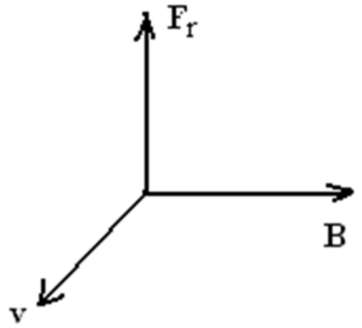
If $\mathbf{E}$ is also present, then $\mathbf{F} = q(\mathbf{E} + \mathbf{v}_\wedge \mathbf{B})$

A charge particle couples to the $\mathbf{E}$ and $\mathbf{B}$ fields with equal strength

Equation (1) gives us the work done in terms of the rate of change of area, using the fact that the field is constant. But the rate of change of area is just the velocity v times the length of the conductor, which is itself the distance through which the force moves the charge to generate the potential difference. Thus we obtain our result, equation (2), which says that the force on a charge q moving with speed v in a magnetic field B is just qvB. The direction of course is along the wire (in the direction of the potential drop). It's easiest to remember the direction from the right hand rule of vector multiplication. If you (or your students) are not familiar with this, another way to get the direction is to use the left hand rule as shown in the figure. Align the first finger of your left hand with the field, your second finger with the speed and your thumb will give you the direction of the force.

Once current flows, there is a force on the wire: this means that the external agent does work. The energy does not come from the magnetic field.

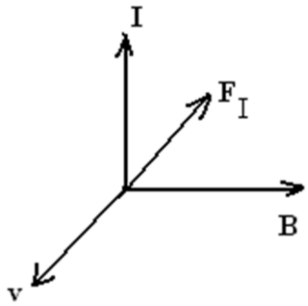
The Lorentz force is perpendicular to the motion of the charge, so does no work. Where then does the energy come from to drive the induced current?



$F_v \bullet v = 0$ Lorentz force does no work

Force on charges in the wire induces a current…

F_I opposes v

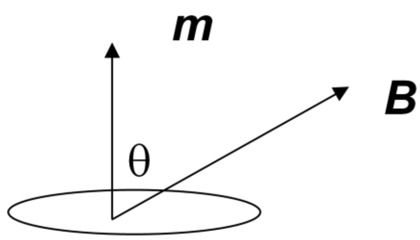


.. and the force on a current carrying wire opposes the motion of the wire

Consider the situation once a current starts to flow: the charges constituting this current now have a motion along the wire. They are subject to a Lorentz force perpendicular to the wire. If you work out the direction you'll see that it is exactly opposite to the motion of the wire. The force on the tether opposes the motion of the spacecraft and tether. This causes the orbit to decay, reducing the gravitational energy of the system. So if we extract power from the tether by drawing a current through it, the energy extracted is paid for by loss in gravitational energy.

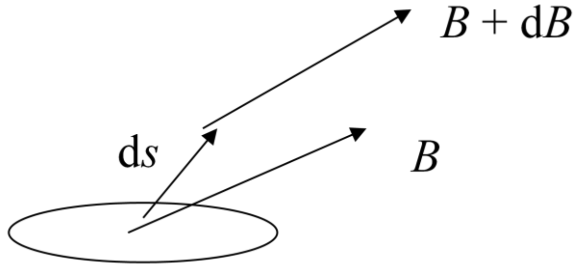
Force on Current loops

However, the tether is part of a current loop and so is the source of the Earth's field. So to be more specific we should look at how the forces act on current loops.



Energy of magnetic dipole $= - \boldsymbol{m}.\boldsymbol{B}$

$= - mB\cos\theta$



Translational Force on loop = - $m\cos\theta\, dB/ds$

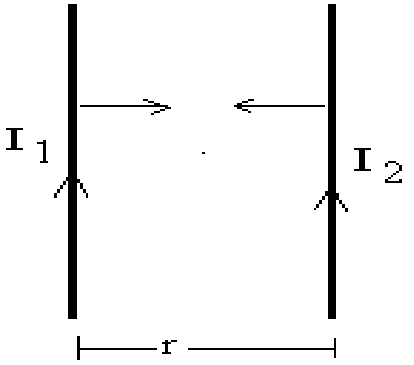
We have seen that we can think of current loops as the elementary components of magnetism, like point charges in electrostatics. As usual the fundamental dynamical quantity is the energy, U say. Once we have this we can derive forces and torques. However there isn't an easy way to derive the energy of a dipole, so we'll just quote the result: the energy of a magnetic dipole aligned at an angle θ to a magnetic field B is mBcosθ or m.B

The force on a magnetic dipole in a direction s will be –dU/ds. The moment m is constant so we get –m.dB/ds for the force. Note this is a bit surprising – the translational force on a dipole in a uniform field is zero; only if the field varies in space will there be a net translational force – although even a uniform field will exert a *couple* on a dipole of mBsinθ.

Torque on loop = $- mB\,d(\cos\theta)/d\theta = mB\sin\theta = \boldsymbol{m} \times \boldsymbol{B}$

Force between Two Wires

Finally, let's look at the force between two wires, because this is part of the definition of the amp. Consider two straight wires a distance r apart.



$$B_{12} = \frac{\mu_o I_1}{2\pi r}$$

Lorentz Force per unit length (“qvB”):

$$\sigma v B = I_2 B_{12} = \frac{\mu_o I_1 I_2}{2\pi r}$$

charge per unit length

The field of wire 1 at wire 2 is obtained from Ampere’s law as in session 5. It is B = $\mu_0 I_1/2\pi r$. The force on a unit length of wire 2 is obtained from the Lorentz la: it is charge per unit length times v times B, or I_2B.

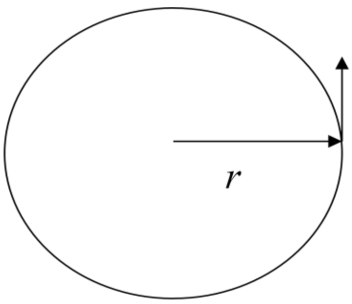
So the force is $\mu_0 I_1 I_2 /2\pi R$. Of course, this is also the force on wire 1 due to the current in wire 2. Note that like currents attract, unlike ones repel.

An amp is defined as the current in each wire a distance of 1 meter apart that gives a force of 1 Newton

Let’s apply what we’ve learnt to the tether. The extraction of power from the tether leads to a force opposing the motion of the tether through the magnetic field. This creates a drag on the satellite-tether system. So this will lead to a decay of the orbit. The ultimate source of the energy is therefore the gravitational potential energy of the satellite.

This suggests that a tether might be used in reverse, as a way of maintaining a spacecraft orbit by driving current through the tether in the opposite sense.

We’re not quite done: suppose we use the system to drive the spacecraft to a higher orbit. What happens to the angular momentum?



$v = (GM/r)^{1/2}$

Angular momentum = mvr

We know that the speed in an orbit of radius r is $(GM/r)^{1/2}$, so v is proportional to $r^{-1/2.}$ The angular momentum mvr is therefore proportional to $r^{1/2}$, hence goes up as the orbit recedes. Where does this angular momentum come from?

It does not come from the Earth's magnetic field. Instead, the magnetic field acts like an elastic connection between the spacecraft and the Earth's core. Pulling on the lines of force at one end eventually moves them at the other causing the core to respond. The source and sink of angular momentum is therefore the rotation of the Earth's core. Looked at another way, the current loop at the spacecraft interacts with the current loop in the core creating a torque on the Earth equal to the rate of change of orbital angular momentum.

The opposite happens when power is extracted from the tether – the Earth acquires the lost angular momentum.

Summary of the Tether Problem

- The cutting of lines of magnetic field by a conductor induces a potential difference between the ends of the conductor
- The circuit can be closed by the plasma (conducting gas) is space
- The magnetic field exerts a force on the current in the tether producing a drag
- The electrical energy produced in the tether therefore comes from the gravitational potential energy of the satellite
- The Earth's dipole field can be thought of as resulting from a current loop
- The current loop at the tether and the equivalent current loop in the Earth's core interact to transfer angular momentum between the satellite and the core.

Section Summary

The force on a charge q moving with speed $\boldsymbol{v}$ in an electric field $\boldsymbol{E}$ and a magnetic field $\boldsymbol{B}$ is $q(\boldsymbol{E} + \boldsymbol{v}\text{x}\boldsymbol{B})$ (the Lorentz Force)

The energy of a magnetic diploe $\boldsymbol{m}$ in a magnetic field $\boldsymbol{B}$ is $-\boldsymbol{m}.\boldsymbol{B}$

The torque on the dipole is $\boldsymbol{m}\text{x}\boldsymbol{B}$

The force between two long straight wires a distance r apart with currents I_1 and I_2 is

$$\frac{\mu_o I_1 I_2}{2\pi r}$$

SAQs

1. A ring of current in a uniform magnetic field suffers a torque but no net translational force. Explain.
   a) The force is always outwards in the plane of the ring
   b) The direction of current flow is opposite on the two sides of the ring so the forces cancel, but the moments do not cancel if the plane of the ring is not perpendicular to the field
   c) There is no force on a ring in a magnetic field

2. Two long straight wires with like currents have the distance between them halved. Does the force (a) double (b) halve (c) quadruple (d) reduce by a quarter (e) stay the same

The answers appear on the following page

Answers

1. a) Wrong: The force is only in the plane if the ring is perpendicular to the field.
   b) Correct: In a uniform field any two elements diametrically opposite each other suffer equal forces, but the moment of these forces about the centre is non-zero if the plane of the ring is not perpendicular to the field.
   c) Wrong: there is no net force because the forces cancel, but each element of the ring that carries a current that is not aligned with the field is subject to a force.

2. a) Correct: this is another example of the fact that not everything is an inverse square law. The force follows from Ampere's law that the field falls off as 1/R
   b) Wrong: The force is stronger if they are closer.
   c) Wrong: it's not an inverse square law!
   d) Wrong: the force increases if the wires are closer
   e) Wrong: assuming that the currents remain fixed. So the force increases by 2.

Additional Problems

Problem 1: Cyclotron

A cyclotron employs a magnetic field to hold charged particles in circular orbits. The size of the orbit is determined by the strength of the field and the energy of the particle. Calculate the radius of a cyclotron required to accelerate protons to 8 GeV in a magnetic field of 9 T.

Orbit equation $\frac{mv^2}{r} = evB$

Energy $E = \frac{1}{2}mv^2$

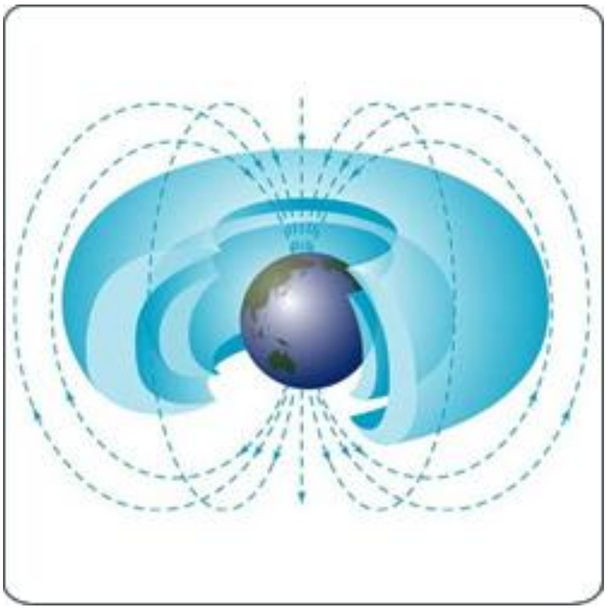
Hence $r = \frac{(2mE)^{1/2}}{Be}$ = 45m

This cyclotron won't actually work. Can you guess why things are not so easy in practice?

To solve the problem we use the Lorentz force law in the orbit equation, and the expression for the kinetic energy to eliminate the speed v. The data are taken from the Large Hadron Collider at CERN, which is 27km in circumference. The discrepancy between the actual diameter and the one we've calculated here arises because we haven't taken relativistic effects into account. This increases the radius by a factor of the square root of (the Energy of the particle divided by twice its rest mass energy)

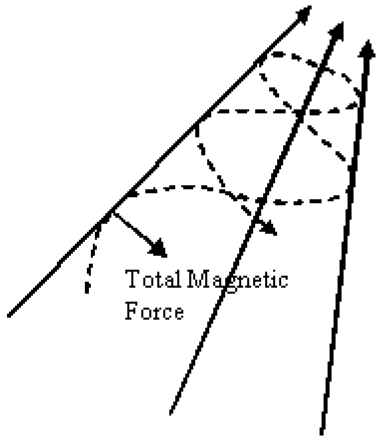
Problem 2: Van Allen Belts

The Earth is surrounded by belts of charged particles trapped by the magnetic field. To do this the field acts like a magnetic bottle. The following images show how this can be understood.



Image[3]

A magnetic bottle:



Consider first the charged particle moving in a plane perpendicular to a uniform magnetic field – the orbit is a circle as in figure 1.

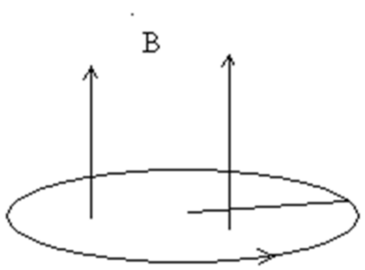


Figure 1

Radius of gyration is given by

$$evB = \frac{mv^2}{r} \text{..................}(1)$$

Mag. force does no work: →energy is conserved

[3] The Location of the Van Allen Belts, BlatentNews.com, posted on www.flickr.com, Creative Commons Licensed.

The radius is given by the orbit equation (1) which balances the magnetic force on the left hand side with the centripetal acceleration on the right. Now add a component of velocity parallel to the field as in figure 2.

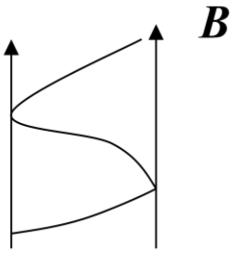


Figure 2 helix

The speed of the motion in the z direction along the helix is constant, since there are no forces in this direction. The charged particle just spirals up the field lines.

Turn now to the case in figure 3 where the converging field lines show that the field is increasing in the z direction.

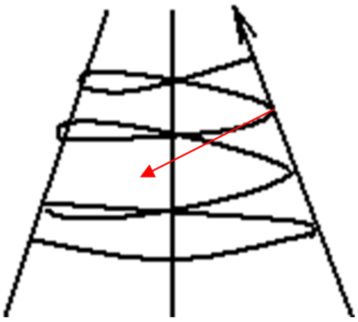


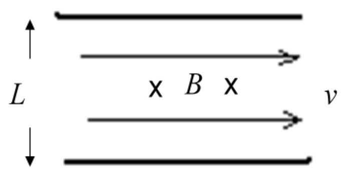
Figure 3

The force is again perpendicular to the field and the velocity, so now there is a component of force in the negative z direction. This reduces the z component and eventually turns it round. This is a magnetic mirror. A magnetic bottle is made from two mirrors as in the shaded regions on the preceding image of van Allen belts. See if you can reproduce the picture of the van Allen belts without looking back.

Problem 3: The English Channel

The English Channel is a conductor flowing through a magnetic field. There is therefore a potential difference between Dover and Calais which is a potential source of power. How large (or small) is it?

We use Faraday's induction law to get the rate at which the sea cuts magnetic flux. The result depends on what you choose for the speed of flow, but the induced voltage is pretty small anyway, and clearly not a viable power source.



$$\begin{aligned} V &= -\dot{\Phi} = BLv \\ &= 0.5\times10^{-4}\times30\times10^{3}\times1 \\ &= 1.5\ \mathrm{V} \end{aligned}$$

Overall Summary

The potential of an electric dipole is given by:

$$\frac{d\cos\theta}{4\pi\varepsilon_0 r^2}$$

The potential of a magnetic dipole is given by:

$$\frac{\mu_0 m\cos\theta}{4\pi r^2}$$

The magnetic dipole moment of a current loop is IA

The energy of a magnetic field B is:

$$\frac{B^2}{2\mu_0}$$ per unit volume

The force on a charge q moving with speed $\boldsymbol{v}$ in an electric field $\boldsymbol{E}$ and a magnetic field $\boldsymbol{B}$ is $q(\boldsymbol{E} + \boldsymbol{v}\mathrm{x}\boldsymbol{B})$ (the Lorentz Force)

Faraday's law: the potential difference induced in a circuit equals the rate of change of magnetic flux linked to the circuit

The energy of a magnetic dipole $\boldsymbol{m}$ in a magnetic field $\boldsymbol{B}$ is $-\boldsymbol{m}.\boldsymbol{B}$

The torque on the dipole is $\boldsymbol{m}\mathrm{x}\boldsymbol{B}$

The force between two long straight wires a distance r apart with currents I_1 and I_2 is

$$\frac{\mu_o I_1 I_2}{2\pi r}$$

Meta tags

Author: Sarah Symons.

Owner: University of Leicester

Title: Enhancing Physics Knowledge for Teaching – Magnetic fields

Keywords: Magnetism; Induction; Forces; sfsoer; ukoer

Description: In this session we'll begin with Faraday's law of electromagnetic induction. We'll describe and use the analogy between a current loop and a magnetic dipole and study the magnetic energy in various situations, including the energy density of a magnetic field. Finally we'll look at magnetic forces from the point of view of the interaction between moving electrical charges, the Lorentz force law, and describe the force between current carry wires, the Biot-Savart law.

Language: English

Version: 1.0

Additional Information

This pack is the Version 1.0 release of the module. Additional information can be obtained by contacting the Centre for Interdisciplinary Science at the University of Leicester. http://www.le.ac.uk/iscience