Session 12

Nuclei and Radioactivity

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Contents

Welcome

Welcome to session 12 of the Physics programme, in which we will look at the nuclei of atoms to explain phenomenon including radioactivity.

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Learning Objectives

- Discuss nuclear stability in terms of binding energies
- Distinguish the different types of radioactivity (α, β, γ), and apply the radioactive decay law
- Distinguish between fission and fusion power
- Explain chain reactions
- Describe how the nuclear force arises from the strong interaction between quarks
- Show a general appreciation of how the standard model of particle physics accounts for the known interactions

The Problem

In the early 1970s, French scientists noticed something odd about samples of uranium recovered from the Oklo mine in Gabon, West Africa. All atoms of a specific chemical element have the same chemical properties, but may differ in weight; these different weights of an element are known as isotopes. Some uranium samples from Gabon had an abnormally low amount of the isotope U-235, which [is crucial for] a chain reaction. This isotope is rare in nature, but in some places, the uranium found at Oklo contained only half the amount of the isotope that should have been there. [Further investigations into this uranium deposit discovered uranium ore with a ^{235}U to ^{238}U ratio as low as 0.440%. This should be compared with the usual concentration of ^{235}U which is 0.7202%.]

From: http://www.science-frontiers.com/sf121/sf121p02.htm

Radioactive Decay

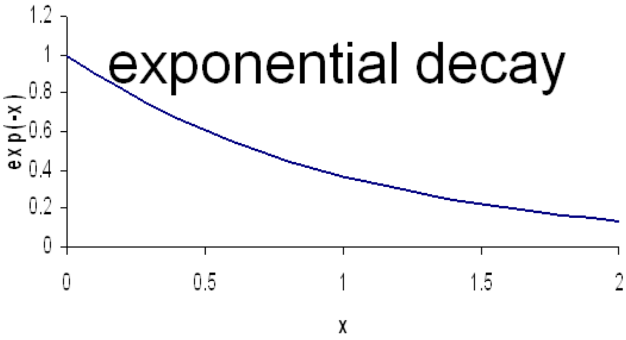
What is radioactivity? Radioactivity arises from spontaneous nuclear transformations in unstable nuclei. You should find it a very strange phenomenon – an individual nucleus behaves totally unpredictably: it may last for years or it may decay after a few minutes. Only the average behaviour is predictable. On average there is a fixed probability of decaying in any given interval of time. This is again different from what we are used to: living organisms are more likely to die as they get older.

Decay Time

This section shows one way of deriving the radioactive decay law without using calculus. It does however require knowledge of the limit expression for the exponential function in the last line.

Probability Approach

This approach gives the exponential decay law directly:



Let the probability of decay in time δt be $p = \lambda\,\delta t$, with λ a constant.

Say we start with N_0 atoms. After one time step we have $(1 - p)N_0$ atoms. After 2 time steps we have $N_0(1-p)^2$ atoms. After n time steps we have $N_0(1-p)^n$ atoms. If these time steps are carried out in time t and have duration δt then $n = t/\delta t$. We arrive at the exponential decay law in the limit as $\delta t \to 0$:

After time t this is $N = N_0(1-p)^{t/\delta t} = N_0(1-\lambda\delta t)^{t/\delta t} = N_0 e^{-\lambda t}$

The Decay Law is: $N = N_0 e^{-\lambda t}$

Why is radioactivity a strange phenomenon from the point of view of classical physics?

Continuum approach:

This is an alternative approach. We model the decay as a differential equation which is solved to give the exponential decay law.

If the decays are independent and random then the number of decays per unit time must be proportional to the number of radioactive atoms N present:

Hence: $\frac{dN}{dt} = -\lambda N$

This implies that the number of radioactive atoms decays exponentially:

$$N = N_0 e^{-\lambda t}$$

With N_0 a constant

Verify this. What physically does N_0 represent?

How Radioactive is a sample?

The activity of a sample is a measure of how radioactive it is – activity is the number of decays per second in the sample. Mathematically for a sample of N atoms the activity is -dN/dt or λN.

Radioactivity: $-\frac{dN}{dt} = \lambda N$

The units in common use for activity are the Becquerel and the Curie.

Becquerel: 1Bq = 1 decay per second

Curies: 1 Ci = 3.7×10^{10} decays per second

Often more useful than the activity is the time that a sample will remain significantly radioactive. If λ is the decay rate of a sample then 1/λ is the characteristic time to decay.

Half Life:

More precisely we use the half-life. The half life is the time taken for half of a sample to disintegrate and is given by $\log_e 2/\lambda$ where λ is the decay rate.

$$0.5N_0 = N_0 e^{-\lambda t_{1/2}} \qquad \text{or} \qquad t_{1/2} = \frac{\log_e 2}{\lambda}$$

See if you can derive these

It can be shown that $1/\lambda$ is the average lifetime of an atom in the sample. To derive this we write down an expression for the mean of the time t and evaluate the integrals which we'll leave as an optional exercise.

$$\text{Mean Lifetime: } \tau = \frac{\int t\,dN}{\int dN} = \frac{\int t e^{-\lambda t} dt}{\int e^{-\lambda t} dt} = \frac{1}{\lambda} = \frac{t_{1/2}}{\log_e 2}$$

Look up the range of half lives (or mean lifetimes)

Summary

- Radioactivity arises from nuclear transformations in unstable nuclei
- The probability of a given nucleus decaying in a given time interval is a constant, independent of time and the behaviour of other nuclei
- The number of nuclei remaining at any time t is $N_0\exp(-\lambda t)$ where $1/\lambda$ is the mean lifetime, and λ is the probability of decay per unit time.
- The half life is $t_{1/2} = \frac{\log_e 2}{\lambda} = \tau \log_e 2$
- Activity, λN, is measured in Bequerels or Curies

SAQs

1. A sample of 1kg of ^{239}Pu has a half life of 24 000 years. What is its activity in Curies to 1 sf?

2. The mean lifetime of an isotope is 130 mins. What is its half life?
   (a) 39 mins (b) 187 mins (c) 90 mins

3. A radioactive tracer is found to have an activity of 8000 counts /min. After 10 mins it is down to 1000 counts/min. What is the half life of the radioactive isotope?
   (a) 289 s (b) 200 s (c) 75 s

4. Neptunium has a half life of 2.14 x 10^6 years. The age of the Earth is 4.50 x 10^9 years. What fraction of the neptunium present at the time of formation of the Earth is present now?
   (a) 0 (b) 0.48 x 10^{-3} (c) 0.33 x 10^{-3}

The answers appear on the following page

Answers

1. 60. Activity = $N\lambda$ = (1kg/239 x mass of H)x$\log_e 2/t_{1/2}$ where $t_{1/2}$ is 24 000 x 3 x 10^7 seconds and mass of H atom is 1.7 x 10^{-27} kg.

2. (a) Incorrect; you have used the incorrect $t_{1/2} = \tau\log_{10}2$ instead of $t_{1/2} = \tau\log_e 2$
   (b) Incorrect: you have used $t_{1/2} = \tau/\log_e 2$ instead of $t_{1/2} = \tau\log_e 2$
   (c) Correct: $t_{1/2} = \tau\log_e 2$

3. (a) Incorrect – you're using the mean lifetime instead of the half life
   (b) Correct – 1000/8000 = $\exp(-\log_e 2\ t/t_{1/2})$ so $\log_e 8 = \log_e 2 \times 10 \times 60 / t_{1/2}$
   (c) Incorrect – the number of counts diminishes exponentially not linearly

4. (a) Correct: the ratio is $\exp(-\lambda t) = \exp(-\log_e 2\ t/t_{1/2}) \sim \exp(-2000)$. This number is so small that even if the Universe were made solely of neptunium when the Earth formed there would be not one atom of it left today.
   (b) Incorrect: you need the exponential of the ratio of timescales $\exp(-\log_e 2\ t/t_{1/2})$ not $t/t_{1/2}$
   (c) Incorrect: you need the exponential of the ratio of timescales $\exp(-t/\tau)$ not t/τ

Radioactive Series

What is an Isotope?

The problem statement talks about isotopes. What is an isotope? Let's take hydrogen as an example. In its commonest form it consists of a single proton nucleus surrounded by a single electron. There are two other forms: deuterium in which the proton in the nucleus is joined by a neutron and tritium in which the proton is joined by two neutrons. Since the electronic structure is the same in all three cases these systems are chemically the same. These different physical forms of an element are called isotopes.

	Z	A	N
Hydrogen nucleus = p	1	1	0
Deuterium nucleus =np	1	2	1
Tritium nucleus = nnp	1	3	2

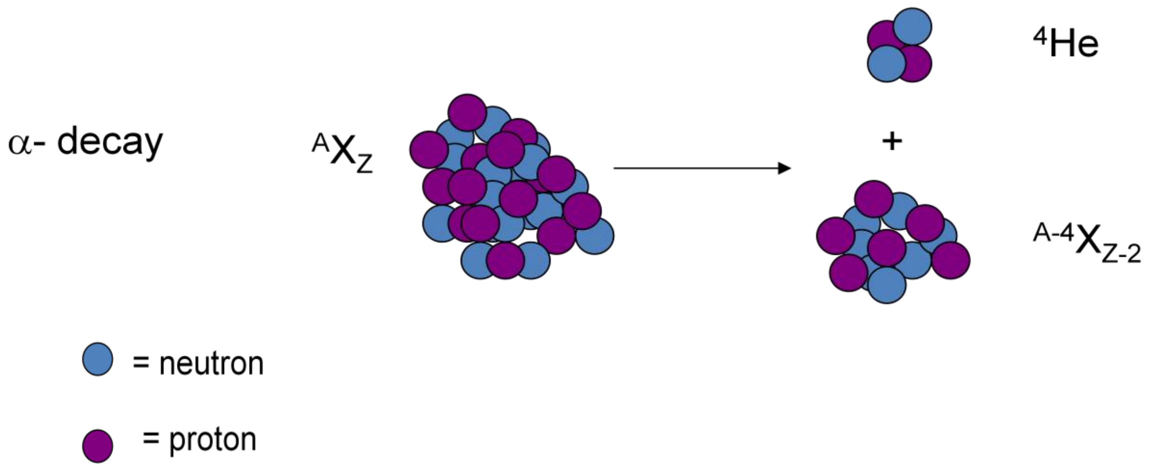
At the other end of the periodic table uranium has 26 known isotopes; but of these only U235 and 238 are sufficiently stable to be found naturally. U 234 is also found naturally as a decay product of U238. (^{234}U, ^{235}U and ^{238}U). The others are made artificially.

How many neutrons in each isotope?

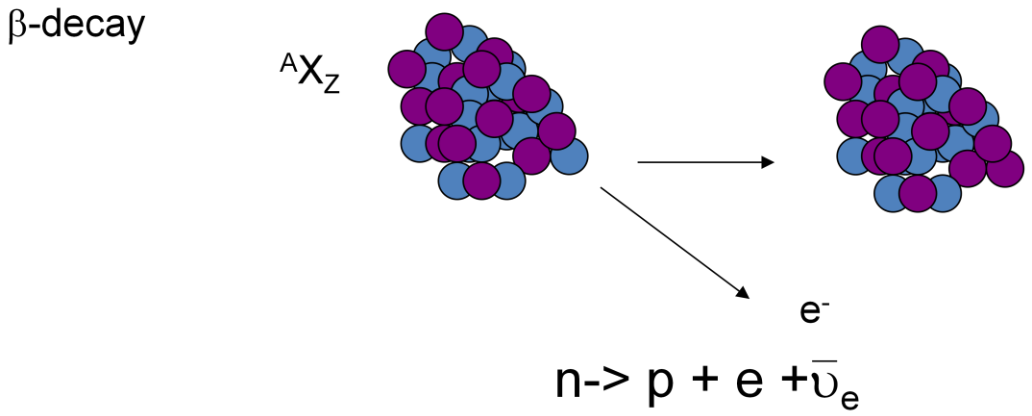
Radioactive Decay Modes

Tritium and all of the isotopes of Uranium are radioactive. So are many of the isotopes of the other elements. But radioactive nuclei decay in different ways. There are two principle modes of decay: alpha decay and beta decay. The section shows what happens in each case.

In alpha decay the nucleus loses a helium nucleus – that is two protons and two neutrons. Historically the helium nucleus in radioactive decays is called an alpha particle.



In beta decay a neutron turns into a proton an electron and another particle with no electric charge called a neutrino (or strictly speaking because neutrinos come in various types, an electron anti-neutrino).



The much rarer inverse beta decay process involves a proton turning into a neutron with the emission of a positively charged anti-electron or positron and an electron neutrino. The product of a radioactive decay is called a daughter nucleus.

For completeness we’ve mentioned gamma decay: this occurs when some other decay process leaves the daughter nucleus in an excited state from which it returns to the ground state by emission of a photon usually in the gamma ray region of the spectrum.

γ-decay Excited nucleus -> ground state

Examples:

The basis of beta decay is that free neutrons are unstable with a half life of about 10 minutes. However, neutrons bound in nucleus decay only if this reduces the mass of the nucleus - we shall return to this later. Examples of beta decay are the decay of Cobalt 60 and Carbon 14 to isotopes of nickel and nitrogen respectively.

Beta decay arises from $n^0 \longrightarrow p^+ + e^- + \bar{\nu}_e$

$$^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni} + e^- + \bar{\nu}_e$$

$$^{14}\mathrm{C} \rightarrow {}^{14}\mathrm{M} + e^- + \bar{\nu}_e$$

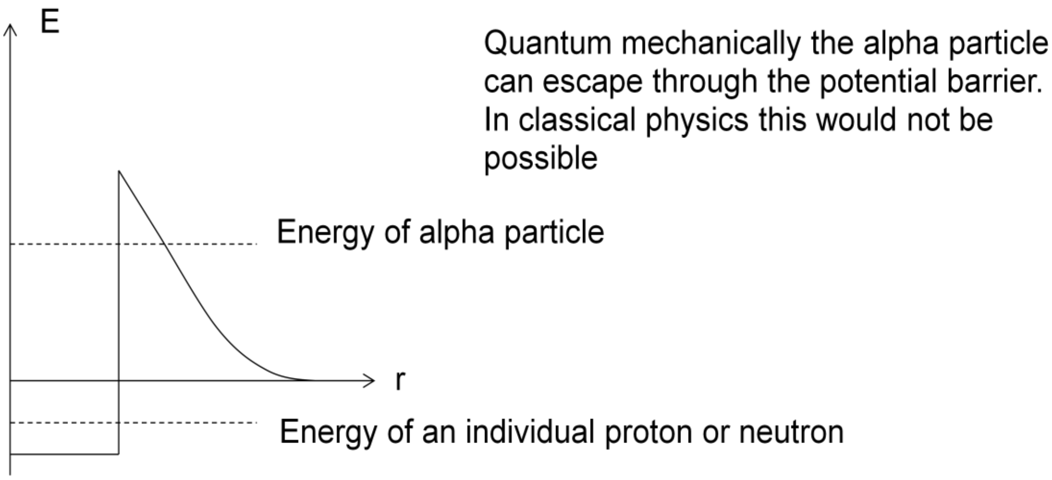
$^{40}\mathrm{K} \rightarrow {}^{40}\mathrm{Ca} + e^-$ 89%

$\rightarrow {}^{40}\mathrm{Ar}$ by electron capture 11%

Some elements can decay in more than one way. These are called branching decays. We’ve shown the example of potassium 40 above.

Alpha Decay

In alpha unstable nuclei the mass of the alpha particle plus that of the daughter is less than the mass of the parent, so the process of decay is energetically possible. However in the nucleus there is a potential barrier holding the alpha particle bound to the nucleus. The figure shows the energy of the alpha in the nuclear potential well.



Quantum mechanically the alpha particle can escape through the potential barrier. In classical physics this would not be possible

Alpha decay therefore involves a process that is not possible in classical physics. But in quantum mechanics there is a probability that a particle can tunnel through an energy barrier and this leads to alpha decay in those nuclei where the overall energetics are favourable.

By subtracting two protons and two neutrons from neutron rich nuclei alpha decay increases the ratio of neutrons to protons in the daughter nucleus.

$$ {}^{A}X_{Z} \;\text{->}\; {}^{A-4}X'_{Z-2} $$

This enhanaces the likelihood of beta decay. Thus alpha-decay produces beta instability so leads to a series of radioactive decays ending with an alpha-stable nucleus.

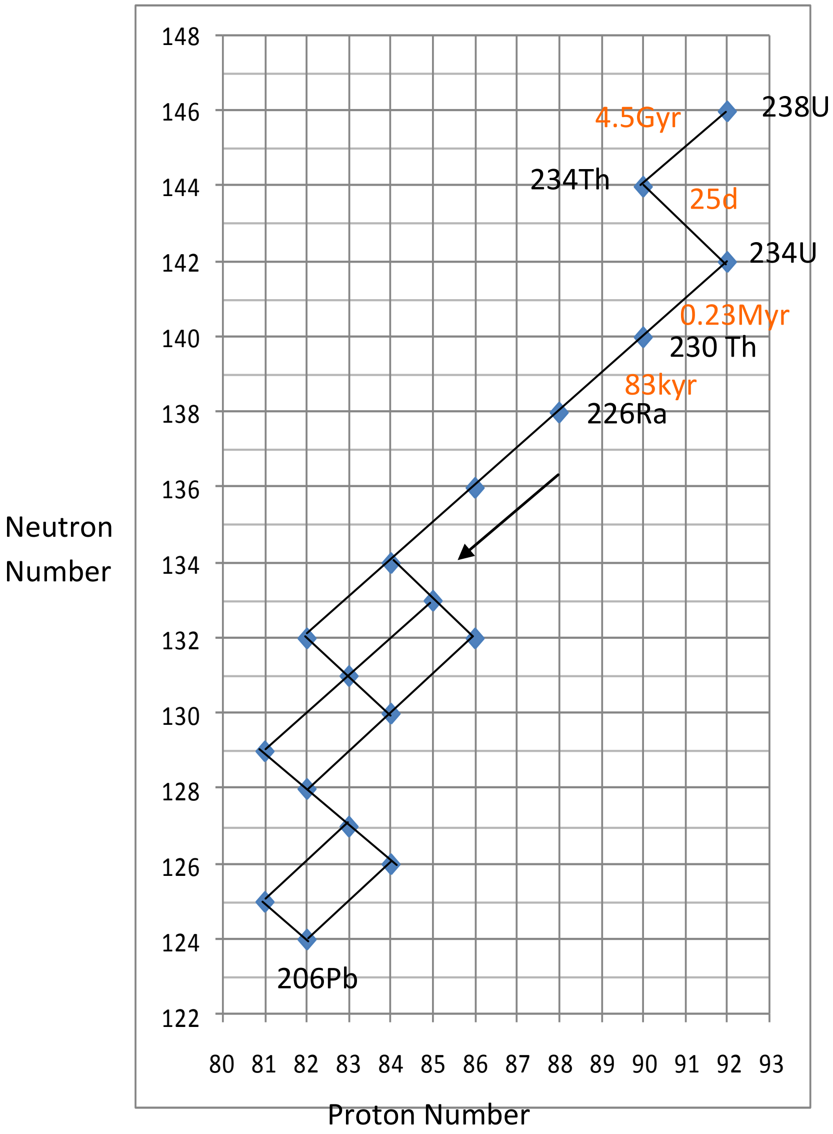
The lifetime rises very rapidly as the energy of the alpha particle falls ie. a factor of 2 to 3 in energy is equivalent to a factor of 10^{24} in half life. This is the Geiger Nuttal law.

The lifetime rises very rapidly as the energy of the alpha particle falls
^{232}Th: 4 MeV alpha; 1.4 10^{10} y half life
^{212}Po: 9.8 MeV alpha; 3 10^{-7} s half life

Why do nuclei emit alpha particles but do not emit single protons or neutrons?

Radioactive Series

This section shows an example of a radioactive series. The first few elements have been labelled, as well as the decay times for each step.



In the U 238 and U 235 series the parent atom has a half life very much longer than that of any of its daughters. So the number of daughter atoms will grow until the rate of decay equals the rate of creation by the parent, that is until $N_p\lambda_p = N_d\lambda_d$. The first daughter decays to a second daughter until $N_{d1}\lambda_{d1} = N_{d2}\lambda_{d2}$ and so on until we reach lead 206, which is stable. So in secular equilibrium, that is in the long term, the Nλ products are equal.

Secular equilibrium: $N\lambda$ = const

$$A \xrightarrow[\lambda_A]{}.\ B \xrightarrow[\lambda_B]{}.\ \dots$$

$$\frac{dN_B}{dt} = -\lambda_B N_B + \lambda_A N_A \quad \text{etc.}$$

In equilibrium this leads to

$$N_A \lambda_A = N_B \lambda_B = \dots$$

Therefore the activities of all daughters equals that of the parent.

How many naturally occurring series are there for U^{238}? Hint: alpha decay reduces the mass number by 4; beta decay does not alter it:

Radioactive Dating with Uranium

Let's look first at dating a rock. We have seen that in long-term or secular equilibrium the λN products are equal for the different decay products. Now, since $\lambda_p << \lambda_d$ the numbers of daughter products in a rock is negligible, typically 0.01% of U 234 and even less of other daughters. Therefore the bulk of the original U 238 in the rock will be in the form of U 238 and lead 206. Thus to a good approximation, once secular equilibrium has been established, we can neglect intermediate stages, and write:

$N_P = N_0 \exp(-\lambda_P t)$.

Then: $N_P(t) + N_0(t) = N_0$

So: $N_D = N_0 - N_P = N_0 (1 - e(-\lambda_P t))$

Or: $$\frac{N_D}{N_D + N_P} = 1 - e^{-\lambda_P t} \qquad (1)$$

We can solve equation (1) for t – the time since the rock solidified and the Uranium became imprisoned.

Why can we ignore the intermediate daughters in dating?

Note that the assumption that the amount of lead 206 present initially is zero is not usually true, but in practice the amount of lead 206 can be obtained from lead 204.

Radioactive Dating with Carbon

Let’s turn next to radioactive dating with carbon (C): Living matter absorbs radioactive 14C from the atmosphere so is in equilibrium with the atmosphere. Dead matter does not have its 14C replenished, so the 14C:12C ratio is a measure of age. This is obtained from the activity of the sample or nowadays by mass spectrometry.

Summary

- Nuclei of the same element may occur with different numbers of neutrons. Such nuclei are called isotopes of the element.
- Radioactive decay modes include alpha and beta decay. Decay with the emission of a gamma ray can occur from an excited nucleus.
- There are four naturally occurring radioactive series
- Measurement of abundance ratios and of activity can be used as a means of dating

SAQs

1. ^{14}C -> X + e +υ_e

   What is X if ν is chargeless and massless?

   a) N 14. (b) B (a) C 13

2. A bone contains 200 g carbon and has a beta decay rate of 400 decays per minute. Find the age of the bone. There are 15 decays per minute per gm of C in a living organism.

   (a) 2368 years (b) 16655 years (c) 11545 years

3. Given the following data, in secular equilibrium what is the concentration of ^{234}U relative to ^{238}U?

   uranium-234: half life = 244 thousand years, 0.0055% of all uranium.
   uranium-235: half life = 704 million years, 0.72% of all uranium.
   uranium-238: half life = 4.5 billion years, 99.28% of all uranium.

   (a) 18.4×10^{3} (b) 5.4×10^{-5} (c) 3.0×10^{-9}

The answers appear on the following page

Answers

1. (a) correct – beta decay changes the mass number by 1 and increases charge number by 1.
   (b) incorrect – beat decay changes a neutron into a proton so moves the atom to the right in the periodic table
   (c) Incorrect – this would imply the loss of a neutron

2. (a) Incorrect – you have forgotten to multiply by the mass of carbon.
   (b) Correct: $400 = 3000 \exp(-\lambda t)$
   (c) Incorrect: you have used the half life instead of the mean lifetime

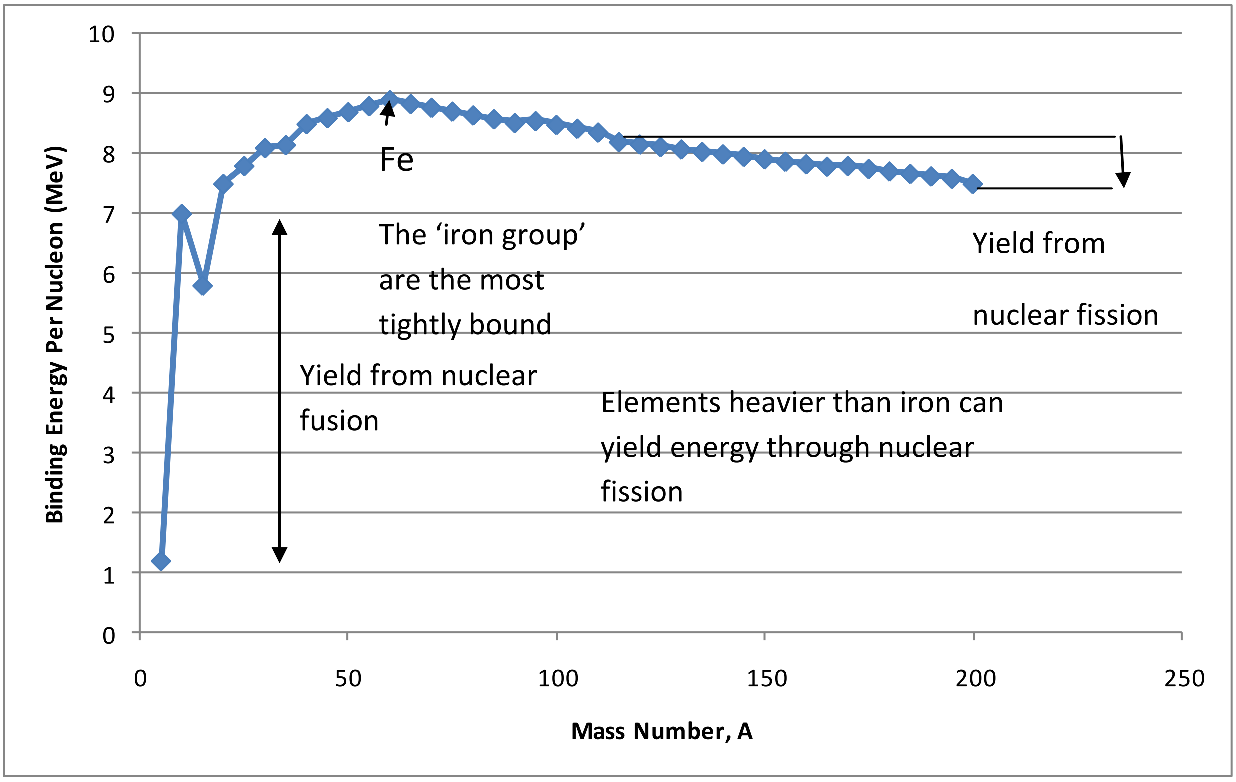
3. (a) Incorrect: you have got the ratio inverted
   (b) Correct: $\lambda_{238} N_{238} = \lambda_{234} N_{234}$
   (c) Incorrect: There is no reason to multiply by the relative abundances

Binding Energy

Why are some Isotopes Stable?

Saturation: energy per nucleon is approximately constant.

An isotope is stable if there are no potential decay products with less mass. Above lead there are no stable isotopes because the masses are greater than the mass of a daughter plus an alpha particle. Hence these isotopes undergo alpha decay.

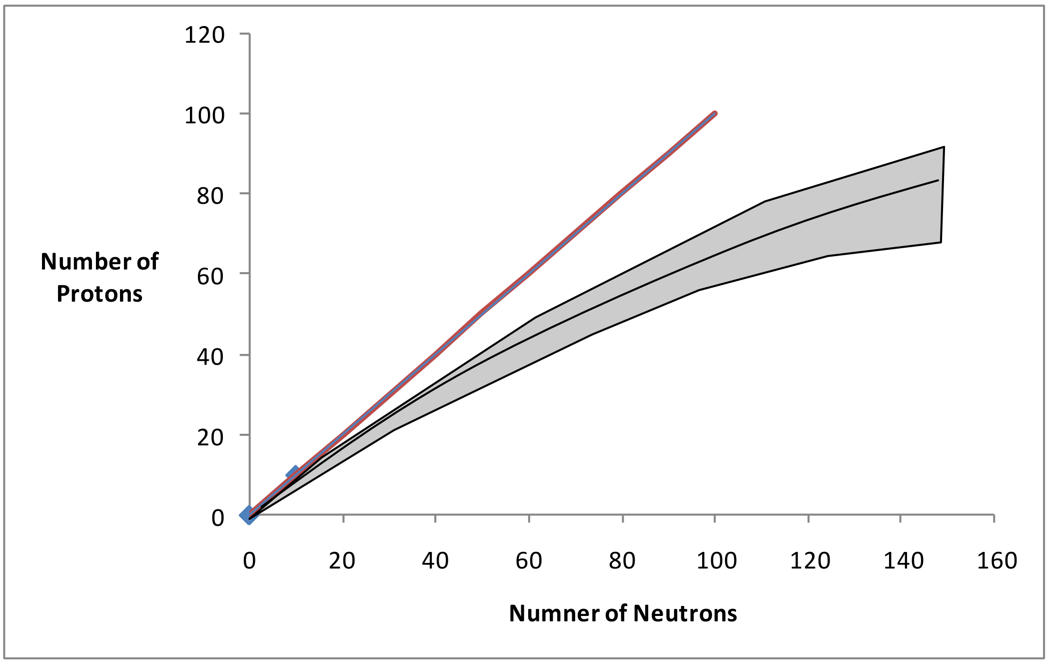


The figure shows the average binding energy per nucleon in a nucleus plotted against mass number: the binding energy is the difference between mc^2 for the nucleus and the sum of mc^2 for the constituents. The figure shows how the binding energy per nucleon changes through the periodic table. With the exception of the region of small mass number, the curve is very flat. This means that nucleons behave as if they can bind with only a fixed number of other nucleons, after which the force is saturated. This is quite different from a force such as gravity where there is no limit on the number of particles that interact – in fact every particle interacts gravitationally with every other. The difference arises because the nuclear force has a limited range beyond which it vanishes, whereas gravity is effective over arbitrarily large distances albeit with diminishing force.

Note the fact that the curve has a maximum means that energy can be obtained by splitting up large nuclei or by combining small nuclei, the basis of fission and fusion power respectively.

Segre plot

In this section we have plotted the neutron number against proton number. It is apparent that stability requires an increasing preponderance of neutrons as the proton number increases. On this diagram, alpha decays move a nucleus along a 45^0 line taking the nucleus away from the line of stability.



The grey area represents naturally occurring isotopes.

The E = mc^2 myth

In this section we'll look at the myth that has grown up around nuclear reactions and Einstein's famous equation, E=mc^2.

The binding energy is c^2 times the difference in mass between the constituents and the whole. This is the energy change (with the appropriate sign) on forming the nucleus or breaking it up.

This leads to the myth that E = mc2 applies only to nuclear reactions and somehow explains where the energy comes from. In fact, it comes from the release of nuclear binding energy, just as the energy released in a conventional explosion comes from chemical binding energy.

i) all systems that emit or absorb energy change their masses according to this law. The formation and break-up of bound systems is associated with an energy change, hence with a change in mass.

ii) Einstein's formula is useful for calculating energy yields for nuclear reactions using data on atomic masses

Summary

- The difference in mass Δm between a nucleus and its separated constituent neutrons and protons is related to the binding energy (BE) by BE = $\Delta m\ c^{2.}$
- Because the nuclear force that holds the neutrons and protons in the nucleus saturates (i.e. each nucleon binds to a fixed number of others) the binding energy per nucleon is approximately constant.
- The binding energy curve shows the departures from constancy due to the Coulomb interaction between the protons.
- Stable nuclei have an excess of neutrons to counteract the Coulomb repulsion of the protons.

SAQs

1. Calculate the energy in MeV to 2 s.f. released in making He^4 from 2 neutrons and 2 hydrogen atoms (enter your answer as an integer).
2. Estimate the total energy in MeV released in the symmetric fission of a ^{236}U nucleus (You can do this from the binding energy curve). State your answer as an integer.
3. If each nucleon in a nucleus were to interact with every other nuclear, the binding energy is approximately proportional to

   (a) A^2 (b) A^3 (c) A

The answers appear on the following page

Answers

1. 28 (accept 25-30). In atomic mass units (amu) $M_{He} - (2m_H + 2\, m_N) = 4.00260 - (2 \times 1.007825 + 2 \times 1.008665) = 4.00260 - 4.032980 = -0.030380$ amu = -28.3 MeV (1 u = 931.5 MeV)

2. 200 (accept 150 – 250) From the BE curve fission releases somewhat less than 1 MeV per nucleon; in symmetrical fission the two daughter nuclei will have about 118 nucleons each so the energy released is around 200MeV (2 fission products x 100 MeV)

3. (a) correct: the number of pairs of interactions is $A(A-1)/2 \sim A^2$ for large A
   (b) incorrect : presumably you are thinking of the increasing volume but this is not directly related to the binding energy
   (c) incorrect: this is how nuclei in fact behave as a result of a short range force that involves interaction with a fixed number of partners.

Chain Reactions

Why doesn’t U explode?

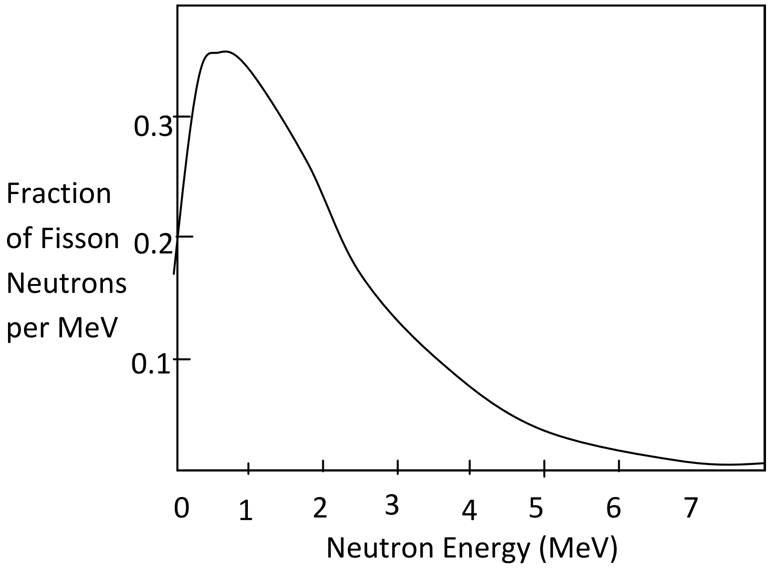
When a neutron collides with a nucleus causing it to split into two usually roughly equal parts the process is called fission. If a number of neutrons are also produced as a result then the process can continue spontaneously.

$$U + n \rightarrow A + B + 2.5\ n$$

U 235 is a fissile material – that is collisions with neutrons of any energy will cause it to split and to produce more neutrons. Why then does uranium not normally sustain a nuclear reaction?

Fission threshold: The neutron must be energetic enough to cause fission. In fact most of the neutrons produced are above threshold. So why doesn’t a large mass of U explode?

In contrast to U235, U238 is non-fissile; that is, there is a threshold energy for the neutrons below which fission cannot occur. Furthermore, U238 is far more abundant than U235, so most of the neutrons will encounter many U238 nuclei before they meet a U235 nucleus. Since however the neutrons produced by fission are above U238 fission threshold, you might think that in bulk natural uranium should in fact sustain a chain reaction. To explain why it does not at present, but was in fact able to in the Oklo mine, we need to study the fate of the fission neutrons in more detail.



What is a cross-section?

In order to understand the fate of the fission neutrons we need to master the concept of a cross section for an interaction. As the name implies the cross section has something to do with area. In fact, for a simple geometrical target it is just the area. In general, a cross section for a reaction is the effective area of the target. In the case of the goal keeper for this example, his effective area is increased over his geometrical area by his mobility.

Cross sections measure effective area for collisions

We've given the effective area of a uranium target nucleus seen by fast neutrons for several types of process, namely fission, absorption and scattering. We'll deal with the two cases of scattering in a moment. Clearly the values of the cross sections tell us how relatively likely the various processes are going to be.

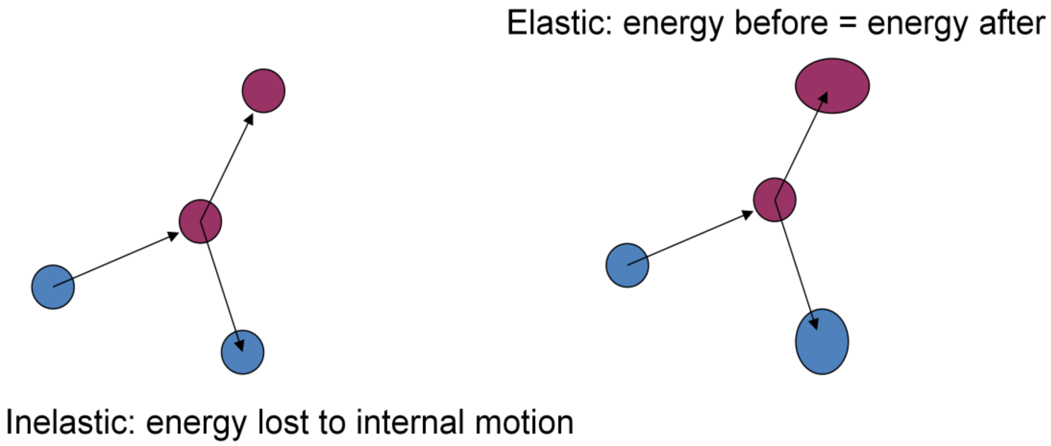
$\sigma_f = 0.5$ b	fission cross section
$\sigma_a = 0.05$ b	absorption cross section
$\sigma_e = 4.55$ b	elastic scattering
$\sigma_i = 2.1$ b	inelastic scattering

The usual unit of cross section in nuclear physics is the barn or 10^{-24} cm^2, which is what the b stands for in the cross sections

What is meant by Elastic scattering

Now let's return to the two types of scattering. In elastic scattering the total kinetic energy of the neutron and nucleus is unchanged – this means that the energy of the neutron is scarcely changed because it is in effect just bouncing off of a much more massive object.

In inelastic scattering kinetic energy is converted to internal energy of the nucleus, which can result in a substantial transfer of kinetic energy from the neutron. The neutrons are therefore slowed down generally to below the fission threshold for U238.



Most of the neutrons undergoing inelastic scattering have their energy reduced below threshold.

The minimum condition for a chain reaction is that each neutron absorbed by a uranium atom must give rise to another neutron.

Let υ be the mean number of neutrons emitted by a fissioning nucleus. For U238 this is typically 2.5. How many further fissions does this give rise to? The ones below the threshold play no role, so we need only consider the fate of those 70% above the threshold. The fraction of these that give rise to fission on encountering a U238 nucleus is a factor of the fission cross section over the total cross section.

First we state: $\frac{\text{fissioncrosssection}}{\text{totalcrosssection}} = \frac{\sigma_f}{\sigma_t}$

If this number is greater than 1 then a chain reaction is possible. We can see that in this case it isn't:

Each fission gives $\frac{\sigma_f}{\sigma_t}\nu$ fast neutrons, of which 70% are above the fission threshold

So each fission gives $0.7\frac{\sigma_f}{\sigma_t}\nu = \frac{0.7 \times 0.5}{0.5 + 0.05 + 2.1} \times 2.5 = 0.35$ new neutrons!

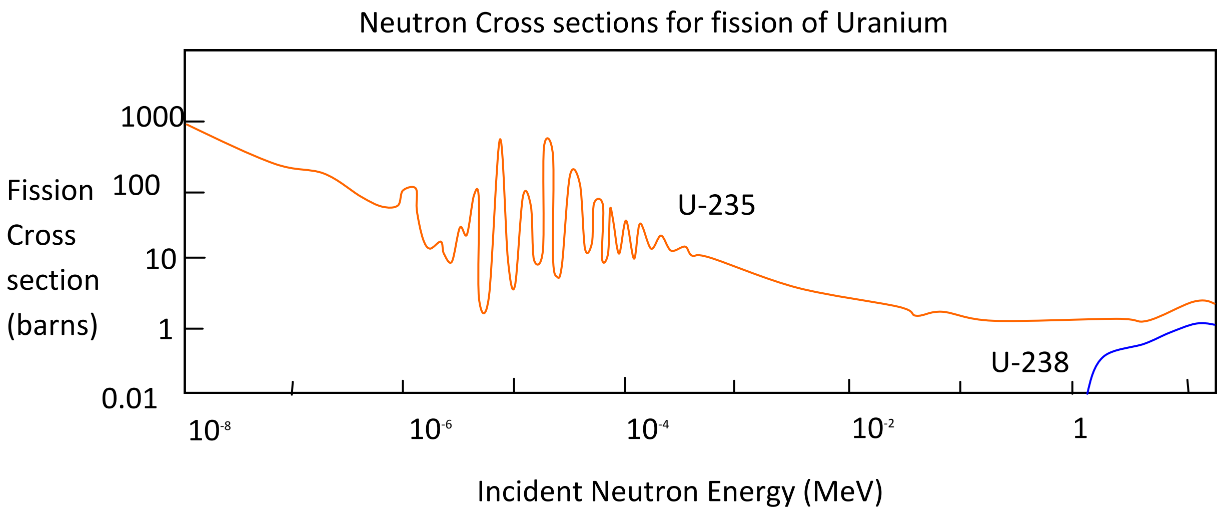
Not enough for a chain reaction

The elastic cross section has been omitted because elastic scattering has no effect on the system – it simply changes the direction of motion of individual neutrons.

Why have we left out the elastic cross section in σ_t ?

So what makes a fission reactor possible?

So what makes a fission reactor with natural uranium possible? The clue lies in the left hand side of this graph which shows that the fission cross sections as a function of neutron energy for U 235 rises steeply at low energies – notice the logarithmic scale to judge just how steeply. In contrast the U238 fission cross section is zero at low energies.



This means that if we can slow fission neutrons down they will interact more with U 235 setting off a chain reaction. To slow down neutrons we use a moderator.

To 'moderate' (that is to slow them down) fission neutrons are remove from the fuel, slowed down by collisions with the moderator and then reintroduced into the fuel. In order to build up a chain reaction we still need to produce more than one neutron per neutron absorbed. To sustain a chain reaction at a given level we need exactly one new neutron for every neutron absorbed in the fuel. Control rods enable this to be achieved in a reactor. Each fission produces on average 2.5 neutrons; not all of these are available to produce further fissions because, once slowed down, they can be absorbed by U 238. The calculation in this section shows that the absorption cross section is sufficiently low that despite the predominance of U 238 more than one fast neutron is produced through fission for each one entering the system. There are some complications of practical detail on the loss of neutrons from the system that reduce the factor of 1.35, but the principle here is correct. A chain reaction is possible using natural uranium as the fuel with a suitable moderator.

Calculation:

Cross sections at thermal energies (~0.25eV): ^{235}U: $\sigma_f = 580$ b, $\sigma_a = 107$ b

^{238}U: $\sigma_f = 0$b, $\sigma_a = 2.8$ b

Why is σf = 0 for 238U here?

For slow neutrons interacting with ^{235}U (0.7% or 1 in 140 U nuclei):

$$\frac{\sigma_f}{\sigma_t}\nu = \frac{580}{580+107+139\times 2.8}\times 2.5 = 1.35$$

1.0 slow neutron -> 1.35 fast neutrons which the moderator slows down to continue the process. This could maintain a chain reaction.

What fraction of neutrons captured by ^{235}U nuclei do not result in fission?

A good moderator will not absorb neutrons and will have light atoms to slow down neutrons efficiently on collision. This is because the collision of a light particle with a heavy particle extracts barely any energy form the light particle. Ordinary water has a relatively large absorption cross section so requires enriched uranium to make an effective reactor; heavy water, that is D_2O is only marginally more massive but has a low absorption cross section for neutrons so is a good moderator, requiring less (or no) enrichment of the natural uranium, but it is expensive.

To make a thermal reactor therefore we need a good moderator. If we use ordinary water as the moderator then too many of the neutrons are absorbed by the hydrogen in H_2O and a chain reaction in natural uranium is not possible. To get a chain reaction with a water moderator requires an increased proportion of U235; that is the uranium has to be enriched. But of course, this is precisely the condition we should have in natural uranium deposits in the past, because the U235 decays more quickly than the U238. We can therefore form a hypothesis about the Oklo reactor that the present depletion of U235 is a result of a natural reactor in the past. To check this we need to work out the enrichment of the uranium at the time of the reactor.

We now show the calculation of the fraction of U235 in natural uranium 2 x 10^9 years ago, using the radioactive decay law for U235 and U238:

Let N_0 be the initial abundances and N the current ones and let λ(5) and λ(8) be the decay constants for ^{235}U and ^{238}U. Then the fraction of ^{235}U at the time of the Oklo reactor ~2 x10^9 years ago is:

$$f = \frac{N_0(^{235}U)}{N_0(^{235}U)+N_0(^{238}U)} = \frac{N(^{235}U)e^{\lambda(5)t}}{N(^{235}U)e^{\lambda(5)t}+N(^{238}U)e^{\lambda(8)t}}$$

$$= \frac{N(^{235}U)}{N(^{235}U)+N(^{238}U)e^{\lambda(8)t-\lambda(5)t}}$$

$$= \frac{0.0072}{0.0072+0.9982\exp(-2.56x10^{-17}x6x10^{16})} = 0.33$$

i.e around 3% - this is consistent

The result is an enrichment of U235 to around 3%, about the same as used in contemporary water moderated nuclear reactors.
This therefore solves the problem.

Summary

- A cross section for a reaction gives the effective area of the target
- 70 % of neutrons produced by fission of ^{238}U are above the fission threshold
- But too many of these are slowed by inelastic collisions to below the fission threshold
- To get a chain reaction we must turn to ^{235}U where there is no threshold and the fission cross section is much enhanced at low neutron energies.
- A nuclear reactor with H_2O as the moderator requires enriched uranium; D_2O moderated reactors can use natural uranium
- The Oklo reactor occurred with enriched uranium (since there would have been a higher percentage of ^{235}U than at present) and with water in the mine as the moderator.

SAQs

1. What are the characteristics of a good moderator?

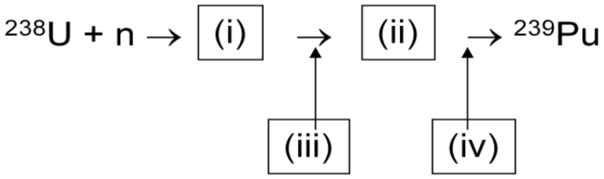
   (a) high absorption cross section for neutrons

   (b) low absorption cross section for neutrons

   (c) high atomic mass

   (d) low atomic mass

2. Fill in the sequence:



   (a) Alpha decay (b) beta decay (c) gamma emission (d) ^{235}U (e) ^{238}U

   (f) ^{239}U (g) ^{239}Np (h) ^{238}Np (i) ^{238}Pu

3. What fraction of thermal neutrons give rise to ^{236}U in a thermal reactor?

   (a) 0.16 (b) 0.1 (c) 0.09

The answers appear on the following page

Answers

1. (a) and (d) Correct: the moderator is there to slow neutrons down, not to absorb them. Collisions between particles of the same mass gives the largest energy exchange, so the atomic mass of the moderator should be as low as possible.

   (b) and (c) incorrect (see above)

2. (i) (f)

   (ii) (g)

   (iii) (b)

   (iv) (b)

3. (a) Incorrect – this takes into account only the ^{235}U; 139 out of 140 neutrons will be absorbed by ^{238}U

   (b) correct – the fraction is σ_a/σ_t for thermal neutrons

   (c) Incorrect – you have used the data for fast neutrons

Additional Problems

Problem 1: The Pu battery of Cassini

The Cassini spacecraft was launched in October 1997 on a voyage to Saturn. It was powered by 33 kg of ^{238}Pu. Although the energy stored in ^{238}Pu is large, the rate at which it can be extracted is limited by the radioactive decay law. How much power was available to the spacecraft on its arrival in 2004 given that the half-life of ^{238}Pu is 88 years and that 238Pu emits an alpha particle with energy 5.5 MeV?

Answer:

We have to work out how many atoms of plutonium were present initially, then how many survived the journey from the radioactive decay law. The number of decays per second times the energy per decay then gives the power available. Note that even though the energy stored in Plutonium is very large, the power available is governed by the radioactive decay rate.

$N_0 = 33 \times 10^3$ kg x 6×10^{23} atoms per mole / 238 kg per mole = 0.83×10^{26}

$$N = N_0 \exp(-t \log_e 2/t_{1/2}) = N_0 e^{-7/127} = 0.95 N_0$$

Power = N/τ x energy per decay = $\dfrac{0.95 \times 0.83 \times 10^{26} \times 5.5 \times 1.6 \times 10^{-13}}{127 \times 3 \times 10^{7}} = 18\,\text{kW}$

Problem 2: Thermal Reactors

Thermal reactors fuelled by uranium release the energy in ^{235}U only. The energy in ^{238}U cannot be liberated directly because it is not fissile at thermal energies, but the ^{238}U can be converted to a fissile material ^{239}Pu. This fissile material can then be separated out and used to fuel another reactor. To sustain a chain reaction requires just over 1 neutron per fission. How many neutrons are required per fission to sustain a breeder reactor which breeds more fissile nuclei than it consumes?

Answer:

Once started the reactor is surrounded by a Uranium blanket from which fissile Plutonium can be obtained as shown below. Two neutrons are required per fission of Plutonium: one to sustain the chain reaction and one to produce another Plutonium nucleus. This stretches the amount of fissile material by a factor of about 60.

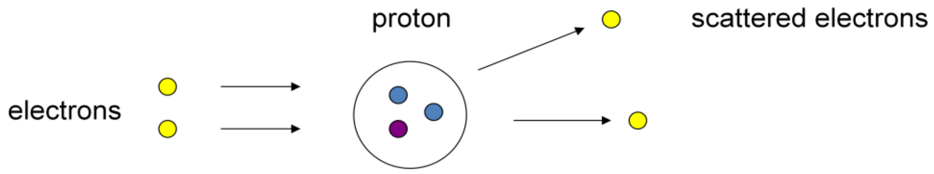
$$^{238}\mathrm{U} + \mathrm{n} \rightarrow {}^{239}\mathrm{U} \xrightarrow[\beta]{} {}^{239}_{93}\mathrm{Np} \xrightarrow[\beta]{} {}^{239}_{94}\mathrm{Pu}$$

When ${}^{239}_{94}\mathrm{Pu}$ undergoes fission it must produce one neutron to maintain the chain reaction and one to create another Plutonium nucleus in the ${}^{238}\mathrm{U}$ blanket. Thus we require at least 2 neutrons. In fact the average number produced per fission is 2.74

Another example of breeding is the production of fissile ${}^{233}\mathrm{U}$ from non-fissile ${}^{232}\mathrm{Th}$

Problem 3: What are nucleons made of?

Our last couple of sections are not so much concerned with a problem you can solve but with the problem of what the ultimate constituents of matter are. How in principle would you find this out – that is how would you find the substructure of a proton? Which of the methods we used in session 11 to find the structure of atoms might apply? One way is to follow the approach Rutherford took and fire high energy particles at nucleons to see how they scatter. This is one of the ways in which the substructure of the proton and neutron were indeed discovered.



“Rutherford scattering” on a proton!

The proton and neutron are found to contain concentrations of fractional electric charge, now called quarks. The next section gives the way in which what is called the standard model classifies the known basic particles of nature from which everything else is constructed.

What does this reveal?

The standard model has three families of particles that make up matter presented here as three rows. They are families in the sense that each column contains particles of similar properties but different masses. There are known to be just three families – there are no more to be discovered.

Fermions: spin ½ particles

Quarks	u (up)	d (down)	Leptons	e (electron)	υ_e (electron neutrino)
	c (charm)	s (strange)		μ (muon)	υ_μ (muon neutrino)
	t (top)	b (bottom)		τ (tau)	υ_τ (tau neutrino)
Charge:	2/3	-1/3		-1	0

proton = uud with spins aligned to give spin ½
neutron = udd with spins aligned to give spin ½

What other spin could a uud particle have?

As well as the particles of matter, there are particles like the photon that are responsible for the forces between matter. The photon carries the electomagnetic force between electrically charged particles – think of radio waves generated in a transmitter and received by you TV aerial. There are similar carriers of the other two forces: the strong force carried by gluons, and the weak force carried by W and Z bosons. The strong force between quarks is responsible for nuclear forces between particles, such as neutrons and protons, made of quarks. The one force that is omitted in the standard model is gravity.

Forces	Bosons (spin 1 particles)
Electromagnetic	photon
Weak	W^+, W^-, Z^0
Strong	gluons

While we cannot go into details, it is possible to convey some idea of how the theory is constructed. Just as we learnt in the sessions on mechanics, we derive the equations of motion of the particles from an expression for the energies of the free particles plus their energies of interaction, essentially the kinetic and potential energies. From this the theory can be used to make predictions about the outcome of decays and collisions which can be checked, very successfully, by experiment.

One final aspect we have omitted: mass in the theory is not an elementary property but itself arises from interactions, in this case with the Higgs particle, the one component in this menu that has not so far been found.

That is: Total energy = energies of free particles (or fields) + energies of interaction

-> equations of motion

-> predictions of bound states and dynamical processes (decay rates, collisions)

+ interaction with Higgs particle to provide masses

Summary

- Radioactive decay takes place at a constant rate per nucleus leading to the radioactive decay law:

$$N = N_0 e^{-\lambda t}$$

- There are two main types of radioactivity: alpha and beta decay
- There are four naturally occurring radioactive series
- A chain reaction can occur in an assembly of uranium lumps in a moderator to slow neutrons to increase the fission probability or in sufficiently enriched uranium
- The Oklo reactor is a naturally occurring example from a time in the past when the relative abundance of ^{235}U was larger (natural enrichment)
- The standard model accounts for the known particles and forces in terms of fundamental constituents

Meta tags

Author: Ted Thomas.

Owner: University of Leicester

Title: Enhancing Physics Knowledge for Teaching – Nuclei and Radioactivity

Keywords: Radioactive decay; Radioactive series; Binding energy; sfsoer; ukoer

Description: In this session we will look at the nuclei of atoms to explain phenomenon including radioactivity.

Language: English

Version: 1.0

Additional Information

This pack is the Version 1.0 release of the module. Additional information can be obtained by contacting the Centre for Interdisciplinary Science at the University of Leicester. http://www.le.ac.uk/iscience