

Session 10

Wave Optics

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Welcome

Welcome to session 10 of the Physics programme. In this session we shall basically be interested in the way visible light exhibits wavelike properties. These properties include the interference of light waves and the diffraction of light round obstacles.

Session Author

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Session Editor – Tim Puchtler

Learning Objectives

- Describe interference and diffraction and show a knowledge of :
 - double slit interference
 - single slit diffraction
 - diffraction gratings
- Explain the diffraction limit to resolving power
- Describe interference application in crystal structure determination (Bragg's law)

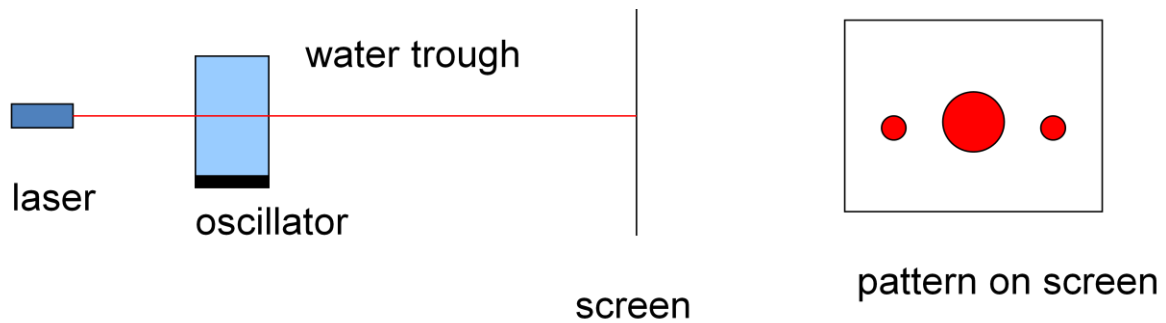
The Problem

The problem for this session is somewhat different from usual. We're asking you to perform an experiment. This is a real experiment in that the equipment responds to your input in exactly the way it would in the laboratory.

Open the experiment (laser-diffraction) from:

<http://open.jorum.ac.uk/xmlui/>

Take a look at the diagram of the experiment. You'll see a laser shining through a tank of water producing multiple spots. The water is driven by an oscillator. You'll find that the spots change as the oscillator dial is turned. How can we explain this? Why does the frequency of the oscillator affect the size and spacing of the spot(s)?



What are our learning issues? The oscillator sets up standing waves in the water. We'll need to find out how light propagates through such a periodic structure. We are going beyond geometrical optics here because clearly the pattern does not arise from light travelling in straight lines: we therefore need to look at the wave properties of light. To make things a bit easier, to start with we'll look at how light propagates through a mask made up of a uniform array of slits, either one or two or many. And to introduce that we'll need to look at how light waves interfere.

You will need to carry out the experiment at some stage. You can of course do this whenever you wish: it may be helpful to wait till we have been through sufficient background to clarify the measurements you need to make; or you may prefer to investigate now how the spot separation depends on the oscillator frequency to have this in mind as a guide to the rest of the session.

Wave Properties and Superposition

Wave Properties: Revision

A wave transfers energy through a medium without the bulk transport of matter. Water ripples are good examples of **transverse** waves in water. They are characterised by how high they sit on the level water surface or how deep the troughs are (**amplitudes**). The separation between troughs is the wavelength.

The time (in **seconds**) it takes for consecutive troughs to pass a fixed point is the **period** of the wave; the number of troughs that pass a fixed point in a second is the wave **frequency**.

Two waves are out of phase temporally (or spatially) if the troughs of one follow those of the other by a fixed time (or distance).

We can represent the amplitude of a wave moving in the +x direction as

$$y = A \sin(\omega t - kx),$$

A is the amplitude, ω is the angular frequency and k is the wave number.

Angular
Frequency
 ω

$$\omega = 2\pi f$$

Frequency
 f

$$f = \frac{1}{T}$$

Period
 T

wave
number
 k

$$k = \frac{2\pi}{\lambda}$$

wavelength
 λ

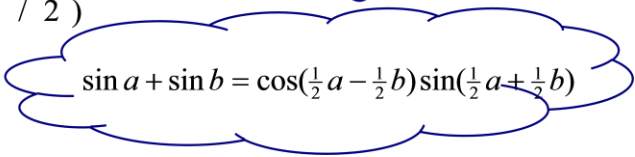
Superposition of Waves

When the two waves y_1 and y_2 are added, the resulting wave will have an amplitude and phase determined by the phase difference, δ , between y_1 and y_2 . We add them together by using the formula for the addition of two sines. If the initial waves each have amplitude A , their sum is a wave of the same frequency and wavenumber, but with an amplitude $B = 2A \cos \delta/2$. If $\delta = 180^\circ$ or π , the sum y_R is 0 and we have total cancellation. We say that we have destructive interference, or that the waves interfere destructively. A phase difference of any other odd multiple of π will also give destructive interference. If $\delta = 0$, the sum $y_R = 2A \sin(\omega t - kx)$: The waves simply add and we have constructive interference. Any other even multiple of π will also give constructive interference.

$$\begin{aligned}
y_1 &= A \sin(\omega t - kx) \\
y_2 &= A \sin(\omega t - kx + \delta) \\
y_R &= A \sin(\omega t - kx) + A \sin(\omega t - kx + \delta) \\
&= 2A \cos(\delta/2) \sin(\omega t - kx + \delta/2) \\
&= B \sin(\omega t - kx + \delta/2)
\end{aligned}$$

where

$$B = 2A \cos(\delta/2),$$



$$\sin a + \sin b = \cos\left(\frac{1}{2}a - \frac{1}{2}b\right) \sin\left(\frac{1}{2}a + \frac{1}{2}b\right)$$

The result is y_R a wave with phase and amplitude determine by δ , the phase difference.

If $\delta = \pi, 3\pi, \dots$ $y_R = 0$: Total cancellation \rightarrow destructive interference

If $\delta = 0, 2\pi, \dots$ $y_R = 2A \sin(\omega t - kx)$: Addition \rightarrow constructive interference

Intensity of Radiation

$$y_R = 2A \cos(\delta/2) \sin(\omega t - kx + \delta/2)$$

The flux of energy carried by a wave – its intensity – is proportional to the square of the amplitude, as we saw in session 3.

$$\text{So for coherent light } I = 4I_0 \cos^2(\delta/2)$$

$$\text{where } I_0 = A^2$$

So the intensity of our two interfering waves is proportional to $4 \cos^2 \delta/2$. Light waves like these, which have a fixed phase relation, are said to be coherent.

Suppose we have two waves to add up, but that their phases are constantly changing at random. The resulting intensity would be the average value of I for the range of phases. To get this we average $\cos^2 \delta$ over all angles which, if we do the integral or look it up, we'll find gives a factor of $1/2$. Thus the addition of two light beams each of intensity I_0 but with a random distribution of phases, which we call incoherent, gives an intensity of $2I_0$, which you might have expected. However you now have two puzzles:

In coherent light the intensity appears to be more than the sum of the constituent intensities: where has the extra energy come from? (if anywhere). The second puzzle arises for incoherent light: if we average the amplitude of y_R we get zero (because the average of $\cos\delta/2$ over all angles is zero). So why doesn't incoherent light cancel to zero rather than give the sum of the energies?

Phase Difference

So far we've discussed adding waves with different phases in rather an abstract manner. How do we actually bring this about? Under what circumstances are coherent waves superposed in the laboratory? Before the advent of the laser as a coherent light source the only way of doing this was to get a light wave to interfere with itself, essentially by splitting it along two or more paths and then bringing the components together. There are two ways of doing this. The first is to divide the amplitude of a beam of light into two beams by a semi-transparent mirror set at an angle to the beam. Half of the light is transmitted, the other half reflected; then the two beams are brought together again. The second method is to divide the wavefront. This is done by passing the wavefront through two or more apertures. The apertures act as a pair of coherent sources.

This is the basis of all interference phenomena. The phase difference in these cases arises from a path difference:

Initially coherent waves travelling along different paths, x_1 and x_2 , to a point P acquire a phase difference at P of

$$\begin{aligned}\delta &= (\omega t - kx_1) - (\omega t - kx_2) = k(x_2 - x_1) \\ &= k\Delta x\end{aligned}$$

If these waves are brought together the resulting amplitude is

$$B = 2A\cos(\delta/2)$$

where

$$\delta = k\Delta x = \frac{2\pi}{\lambda}\Delta x$$

Wave Superposition

The exhibition of interference then depends on setting up the path difference so that $k\delta x$ is an appropriate phase difference for interference.

Generally

for $\Delta x = (m + \frac{1}{2})\lambda$, or $\delta = (2m + 1)\pi$

→ destructive interference, ($m = 0, 1, 2, 3 \dots$)

for $\Delta x = m\lambda$, or $\delta = 2m\pi$

→ constructive interference

Two waves of the same frequency and wavelength will have a phase difference at a point as a result of a difference in path length

Two such waves having a constant phase difference at all times are said to be coherent waves.

In general, interference cannot be observed from the superposition of incoherent waves.

Summary

- For a wave of the form:

$$y = A \cos(\omega t - kx)$$

- A is the amplitude, $\omega = 2\pi/f$ the angular frequency, and $k = 2\pi/\lambda$ the wavenumber
- For a path difference Δx the phase difference is $k \Delta x$

SAQs

1. The phase difference between the waves

$$y = A \cos(\omega t - kx) \quad \text{and} \quad y = B \sin(\omega t - kx)$$

is:

- a) 0
 - b) π
 - c) $\pi/2$
 - d) $-\pi$
 - e) $-\pi/2$
2. The phase difference between two wavefronts a distance Δx apart is
- a) 0
 - b) Δx
 - c) $2\pi\Delta x/\lambda$
 - d) $-2\pi\Delta x/\lambda$

The answers appear on the following page

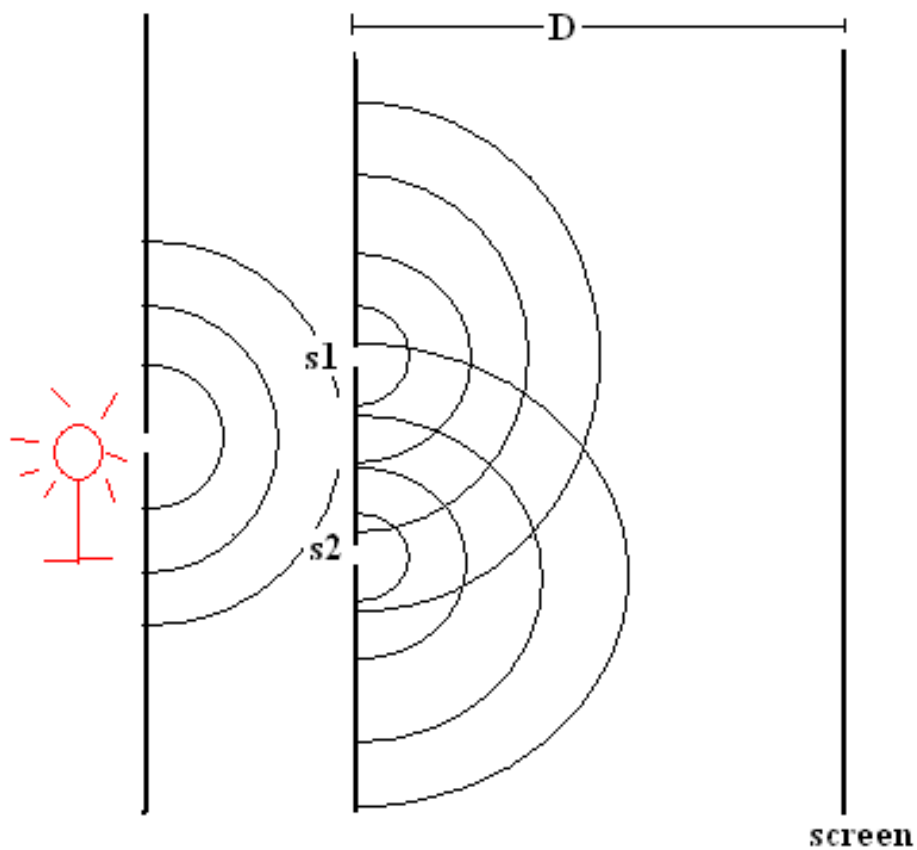
Answers

- a), b), c), d): Incorrect: cos lags sin by $\pi/2$; mathematically $\cos(\theta - \pi/2) = \sin\theta$
e) Correct: $\cos(\theta - \pi/2) = \sin\theta$
- a) Incorrect: to traverse a distance Δx takes a time $\Delta x/c$ so $\omega\Delta t - k\Delta x = ck\Delta x/c - k\Delta x = 0$; but what is required is the the phase difference at a fixed time
b) Incorrect: the phase difference is the angle turned through in the distance Δx , not the geometrical distance
c) Incorrect: this is the right magnitude but strictly speaking the wrong sign; $(\omega t - k(x+\Delta x)) - (\omega t - kx) = -k\Delta x = -2\pi\Delta x/\lambda$
d) Correct: the phase difference measured in the direction of the wave is $(\omega t - k(x+\Delta x)) - (\omega t - kx) = -k\Delta x = -2\pi\Delta x/\lambda$

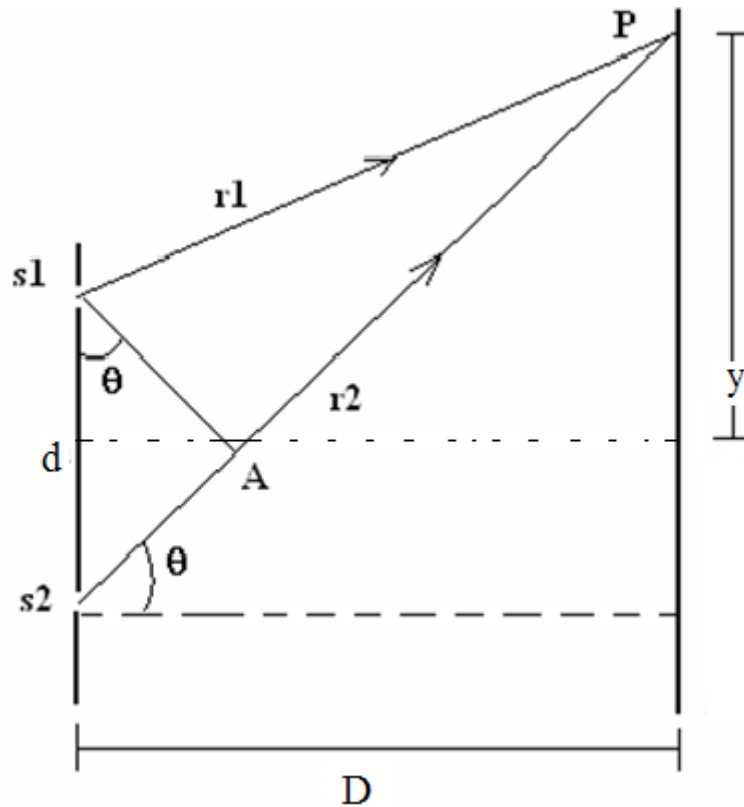
Interference

Huygens' Principle

When light is admitted to two very narrow slits, each slit serves as a 'secondary' source of coherent light waves. This is Huygens' Principle. When observed on a distant screen, alternate bright and dark patterns (fringes) can be seen. This is a result of interference of the 'secondary' waves. One can understand the interference by considering the path difference between pairs of waves at the screen.



Wave Interference



This is the basic set-up for interference: the two very narrow slits \$s_1\$ and \$s_2\$ act as point sources. We assume they are coherent, for example because they originate from a single source, and monochromatic. The illumination at \$P\$ is formed by the interference of light rays from each of the two sources. The path difference will determine whether this is constructive or destructive.

Since the angle at \$P\$ is very small, the path difference \$r_2 - r_1\$ is close to \$d \sin \theta\$.

$$\Delta r = s_2 A = r_2 - r_1 = d \sin \theta$$

For constructive interference (bright band):

$$\delta = k d \sin \theta = 2m\pi \quad (m=1,2,3)$$

Or, since \$k = 2\pi/\lambda\$:

$$d \sin \theta = m\lambda$$

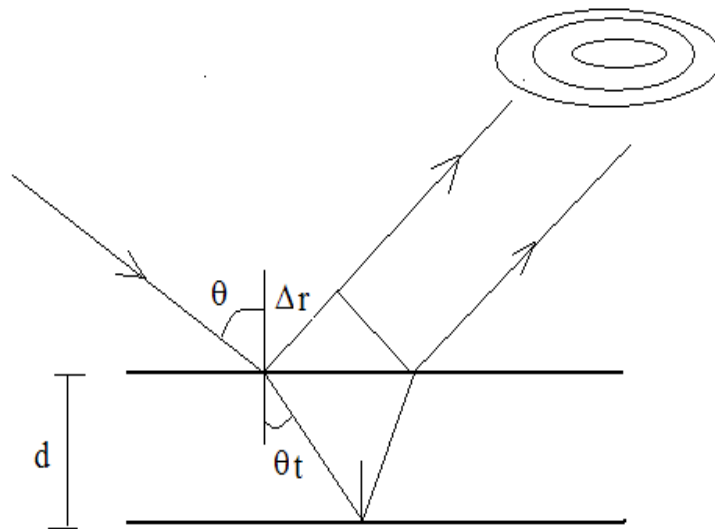
Similarly for a dark band (destructive interference):

$$d \sin \theta = (m + \frac{1}{2})\lambda$$

To summarise, the phase difference at P is $\delta = kd\sin\theta = 2m\pi$ for a bright spot. Since $k = 2\pi/\lambda$ this gives the condition $d\sin\theta = m\lambda$ for an intensity maximum, where m is an integer. Similarly for a dark band $d\sin\theta = (m+1/2)\lambda$.

Interference maxima occur for $d\sin\theta = m\lambda$

If the amplitude of a wave is split among secondary waves while the phase difference is preserved, then interference can be observed when they are recombined. Waves can be split by reflection and transmission.

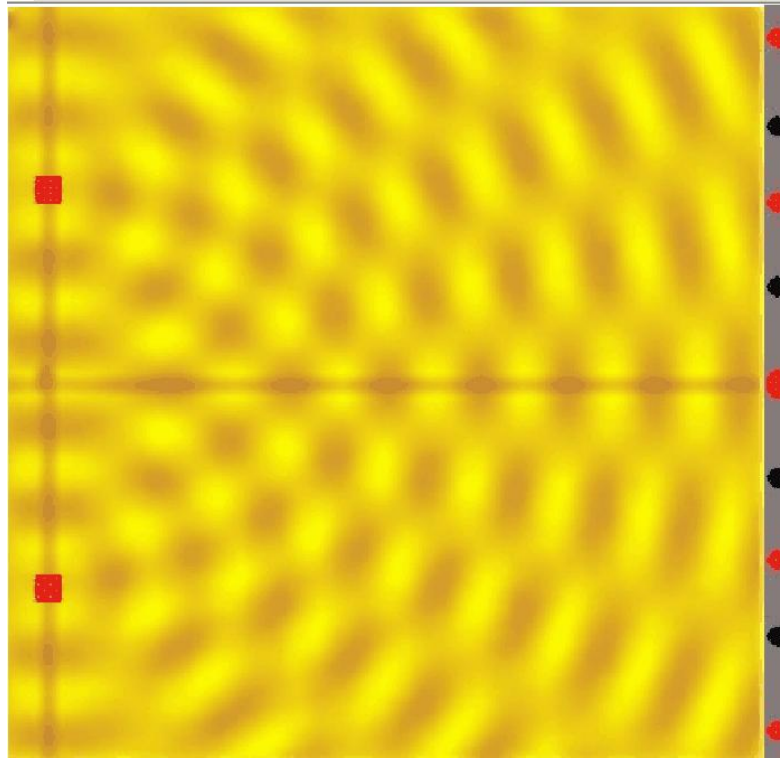


For example, light reflected off the first and second surfaces of a thin film produces interference fringes if the path difference between the reflected rays is such that the interference condition is satisfied. In this case the m th interference order relates the thickness, d , the refracted angle, θ_t , and the wavelength λ_f in the film as:

$$d \cos\theta_t = (2m+1) \frac{\lambda_f}{4}$$

Examples of Interference:

Thomas Young's sketch of two-slit diffraction (from around 1800).



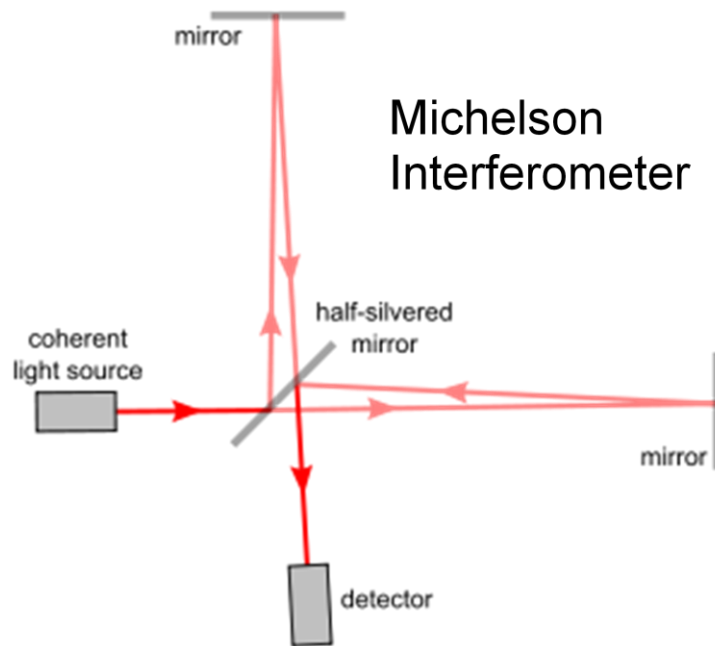
Image¹

Thomas Young was a champion of the wave theory of light on the basis that it could account for interference phenomena. It should be easy to understand how waves on the surface of a pond add when they are in phase and cancel when they are 180° out of phase.

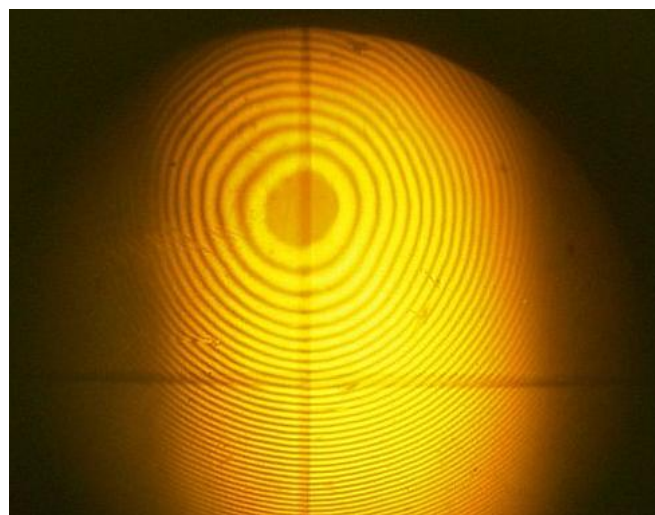
You can repeat the experiment to some extent by looking at the pattern formed by dropping two stones into a pond.

The Michelson interferometer can be used for example to disperse light to obtain a line spectrum.

¹ Young_Experiment, Qatar, as posted on commons.wikimedia.org. Public Domain.

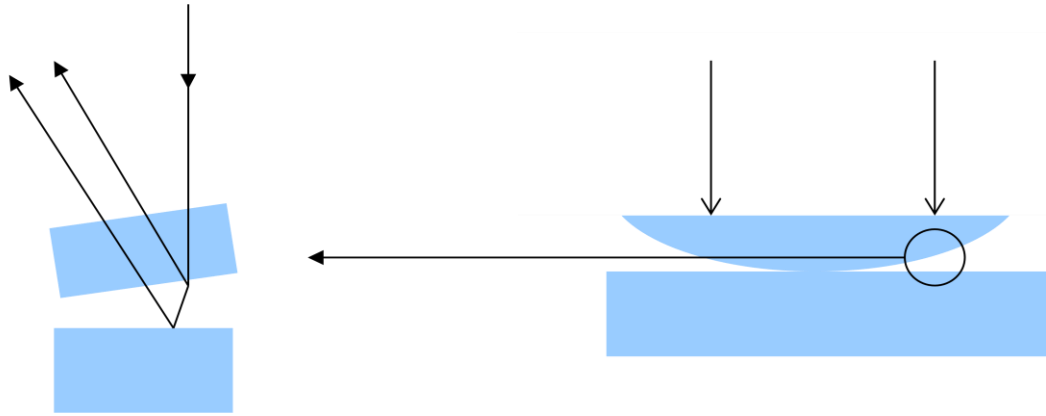


Newton rings are an example of interference that can be produced fairly readily in the laboratory. The condition for a bright fringe is somewhat different from what we have seen so far, because there is an extra phase change of 180° on reflection from the interface between the glass and air (but not between the air and glass). This adds an extra $\lambda/2$ to the path difference so constructive interference first occurs when the geometrical path difference is another $\lambda/2$. In general reflection from the interface between a medium of high refractive index and one of lower refractive index adds a phase change of π .



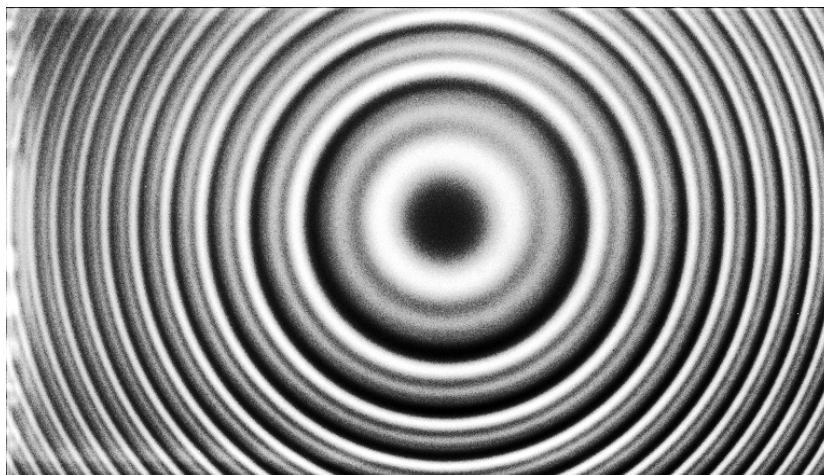
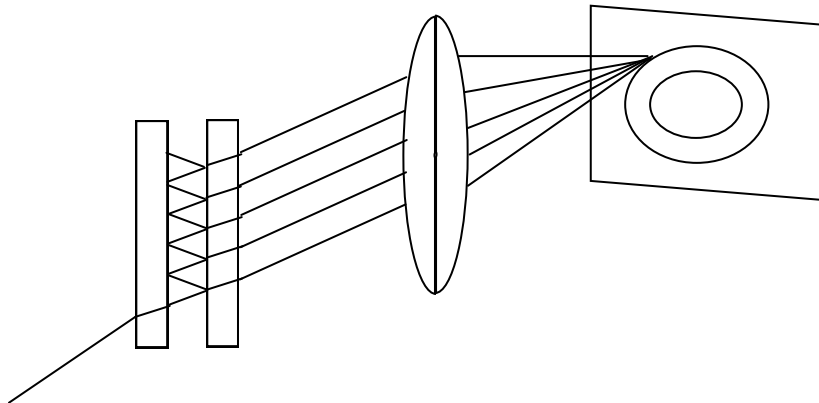
Image²

² Newton Rings, by Winnie Summer, as posted on commons.wikimedia.org. Creative Commons Licensed.



A diagram depicting how Newton's Rings form.

The Fabry Perot interferometer uses multiple reflections in a thin plate to create a sharp interference pattern.



Image³

³ Fabry_Perot_Etalon_Rings_Fringes, by John Walton, as posted on commons.wikimedia.org. Creative Commons Licensed.

Periodic Structure in the laser experiment

How far has this taken us towards understanding our experiment? The laser is a coherent light source so interference is a possibility. The pattern of dots depends on the oscillations in the water, so it is not due to interference from reflections in the glass container.

The oscillator sets up a standing wave pattern in the water. The density of water changes ever so slightly with compression and rarefaction – and this changes the refractive index. So we have a periodic structure. On what scale is this periodic? We need to know if the laser beam intersects one period or many.

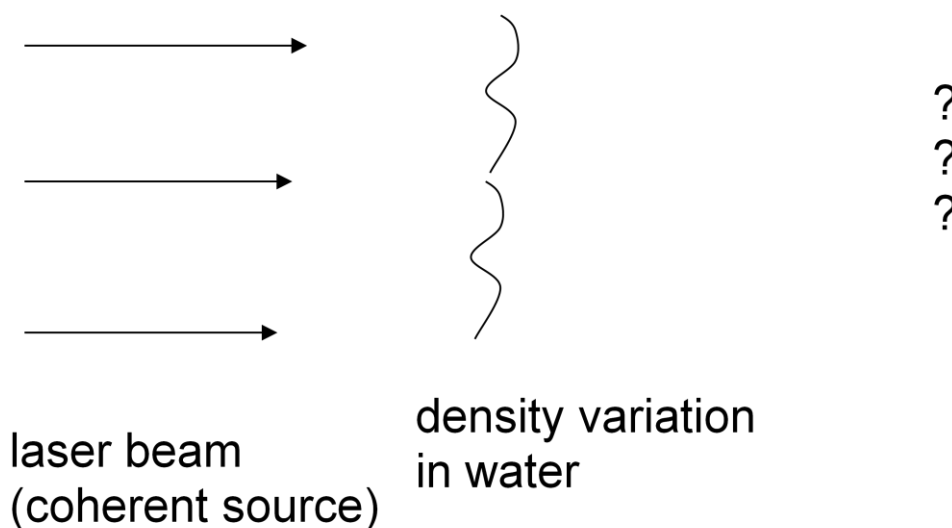
The compressions and rarefactions in the water are just sound waves. So the periodicity is just the wavelength of the sound waves in the water. We can calculate this from the frequency and the speed of sound in water.

$$\text{Wavelength in water} = \text{speed of sound} / \text{frequency}$$

Work out the periodicity of the refractive index in the water.

Does the laser beam intersect many periods?

We get 0.1 mm. What is the significance? The laser spot with no interference is of the order of a centimeter, so it intersects many wavelengths of the sound wave structure. Thus we need to think about interference from not just one slit but many – and also the slits aren't so thin that we can use the point source approximation. This brings us to diffraction, which we'll look at in the next section.



Note: at 25 degrees C changing the pressure from 1 bar to 10 bars changes the density from 997.05 to 997.45 kg/m³, and this changes the refractive index (sodium-D line) from 1.33286 to 1.33299 so for a 1% change in pressure get about 10⁻⁷ change in refractive index.

Summary

- Interference occurs when coherent sources are added
- If two sources differ in phase by δ the intensity of the combined wave is proportional to $\cos^2 \delta / 2$.
- For waves with different path lengths $\delta = k\Delta x$, where k is the wave number
- Huygens' principle states that each point on the wavefront is a source of secondary waves. This can be used to calculate the path lengths.
- For two slit interference the maximum intensities of the fringes are located at angles given by $d\sin\theta = m\lambda$.

SAQs

1. Which of the following are coherent sources?
 - a) a laser
 - b) a torch
 - c) a radio galaxy
 - d) a microwave oven
2. Interference can occur between waves of different frequency as long as they are coherent – true or false?
3. If the two slit interference experiment is carried out with white light what would the fringes look like?
 - a) there would be no fringes because waves of different frequency cannot interfere
 - b) there would be no fringes because white light cannot be coherent
 - c) the fringes would be in black and white
 - d) the fringes in different colours would be displaced and would overlap
4. What is the minimum path difference that will produce a phase difference of 180° for light of wavelength 500nm? Give your answer in nm as an integer.

The answers appear on the following page

Answers

1. Coherent:
 - a) although the light is emitted from many atoms these are made to emit in phase in the laser
 - c) because the sources are at such large distances the radiation is spread out over a large surface. This means that the wavefronts arriving at the Earth are coherent over measureable distances and times. This is used to determine the direction of cosmic radio sources

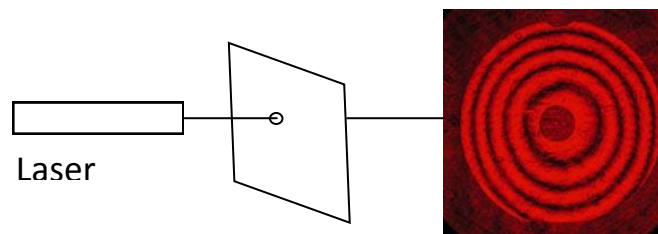
Not Coherent:

 - b) the torch beam is generated by many atoms emitting short wave trains entirely independently, so it is not coherent.
 - d) as for the torch beam, the microwaves are generated independently
2. False: waves of different frequency cannot remain coherent i.e. cannot oscillate together, since they have different periods.
3.
 - a) Incorrect – the interference is between waves of the same frequency in the white light
 - b) Incorrect – as long as the interfering beams originate ultimately from a single source they will be coherent
 - c) Incorrect – a reasonable guess, but not if you understand that the location of the interference pattern on the screen depends on the wavelength of the light. It must therefore be different for different wavelengths
 - d) Correct – the interference pattern on the screen depends on the wavelength of the light and must therefore be different for different wavelengths
4. Correct: 250nm
Incorrect: Have you converted 180 degrees to radians – the formula for path difference requires δ to be in radians.

Diffraction

When a light wave is obstructed by an object we generally draw a shadow divided into umbra and penumbra. For small enough objects however, a bright spot is observed at the centre of the shadow as well as rings around the shadow.

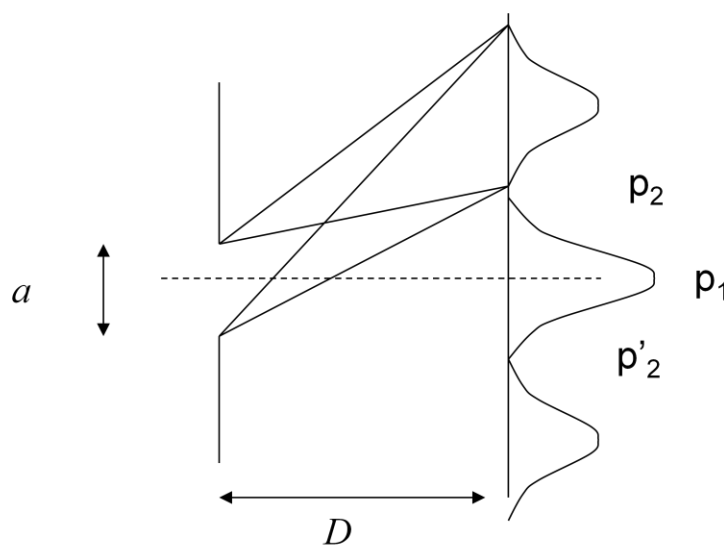
This behaviour can be traced to the fact that on scales of order a few wavelengths the wave properties of light have to be taken into account, and geometrical optics is not an adequate approximation. The converse phenomenon is observed on shining light through a circular aperture – the result is not a bright spot but a diffuse disc, known as Airy's disc which is surrounded by concentric rings of decreasing brightness.



Image⁴

Diffraction

Diffraction is the bending of waves on obstruction and the resultant interference pattern beyond the obstructed region.

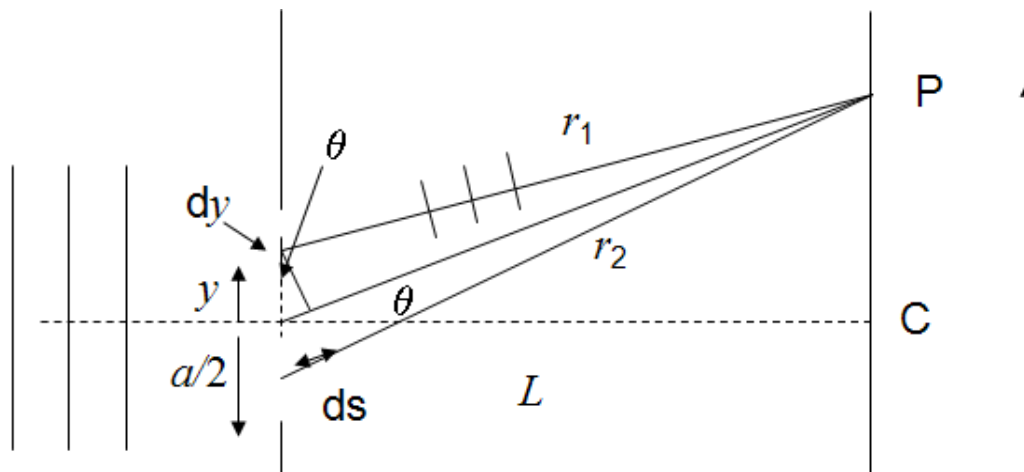


⁴ Diffraction Pattern Image: 'new_igramm_2', by Philipp Salzgeber, as posted on www.salzgeber.at. Creative Commons Licensed.

For a 'not so narrow single slit' a bright band is registered opposite the centre of the aperture. On either side of the aperture, rays arriving at an angle interfere. Dark bands occur at points p_2, p_2' where the path difference ($a \sin \theta$) between rays at the top and lower edge of the aperture are integral multiples of λ . This is because, as we'll see in a moment, pairs of rays between these arrive $\lambda/2$ from each other, i.e. out of phase by π and destructively interfere. In general dark fringes occur at angles given by $a \sin \theta = m\lambda$.

Diffraction Amplitude

Now we'll work out the location of the diffraction maxima and minima. The calculation is similar to that for the interference pattern. The difference is that instead of two point sources we have an aperture of sources each interfering with all the others. The aperture acts as a source of secondary rays for which we calculate the path differences. Of course, the source has to be coherent across the aperture.



Let's look at the point P on the screen. The simplest way to look at this is to think of pairs of sources interfering, one from an element dy at y and one from an element dy at $-y$. We can then use what we have learnt about the superposition of two waves.

The rays from the element of aperture dy at the point y from the centre have path length r_1 . From the aperture dy at $-y$ in the lower half the path length is r_2 . From the small triangle, the difference $r_2 - r_1$ is approximately $2y \sin \theta$. Thus, the phase difference is $\delta = 2ky \sin \theta$.

$$\delta = k(r_2 - r_1) \approx 2ky \sin \theta$$

The contribution to the amplitude from the element dy is therefore $dA_P = 2A \cos(ky \sin \theta) dy$ from section 1.

$$dA_p = 2A \cos(\delta/2) dy = 2A \cos ky \sin \theta dy$$

To get the total amplitude at P we integrate over y between the limits 0 and a/2 since this will take in the whole aperture. This gives us equation (1).

$$A_p = \int_0^{a/2} 2A \cos ky \sin \theta \, dy = \frac{2A}{k \sin \theta} \sin\left(\frac{ka}{2} \sin \theta\right) \quad (1)$$

Diffraction Intensity

We rewrite our expression for the amplitude in terms of a quantity $\beta = ka/2 \sin \theta$ as in equation (1).

$$A_p = \frac{2A}{k \sin \theta} \sin\left(\frac{ka}{2} \sin \theta\right) = \frac{4aA}{\frac{ka}{2} \sin \theta} \sin\left(\frac{ka}{2} \sin \theta\right) = \frac{4aA}{\beta} \sin \beta \quad (1)$$

where $\beta = \frac{ka}{2} \sin \theta$

The interference diffraction pattern is obtained by squaring the amplitude giving us equation (2).

$$I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \quad (2)$$

Where I_0 is the amplitude at the centre.

Here we have simply defined I_0 to be the intensity when $\theta = 0$. The diffraction minima occur when $I = 0$ at $\sin \beta = 0$. This gives the diffraction condition $a \sin \theta = m\lambda$ for the minima.

Diffraction minima occur for $a \sin \theta = m\lambda$

Diffraction Grating

A diffraction grating is a set of equally spaced slits which may transmit or reflect light. A CD is a common example of a reflection grating. The multiple slits create a sharper fringe pattern. The diffraction grating is quite complicated because it combines diffraction and interference: the diffraction at each of the slits of width 'a' and the interference between the

light from these slits with separation d . Thus in general we get a diffraction pattern superimposed on an interference pattern. A diffraction grating has many slits typically 100 slits per cm of surface.

The grating equation for the interference maxima is unchanged:

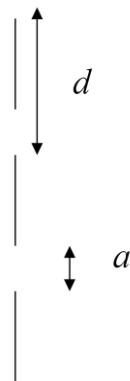
$$d \sin \theta = m\lambda$$

The diffraction minima occur at:

$$a \sin \theta = m\lambda$$



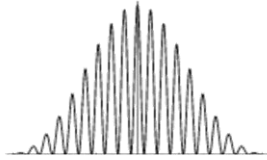
Image⁵



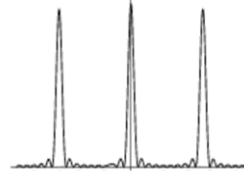
$d = \text{separation}$
 $a = \text{slit width}$

⁵ ubuntu-cd, by --b--, as posted on www.flickr.com. Creative Commons Licensed.

We now show two examples of diffraction and interference. The intensity pattern in (a) arises from two slits – the interference peaks occur at angles given by $d\sin\theta = m\lambda$ and the diffraction envelope has its first minimum at an angle given by $a\sin\theta = n\lambda$. Increasing the number of slits emphasises the diffraction peaks clearly in (b), although the ripples of the interference can still be made out.



(a) Diffraction pattern superimposed on the interference pattern for two slits, separation 5 x width

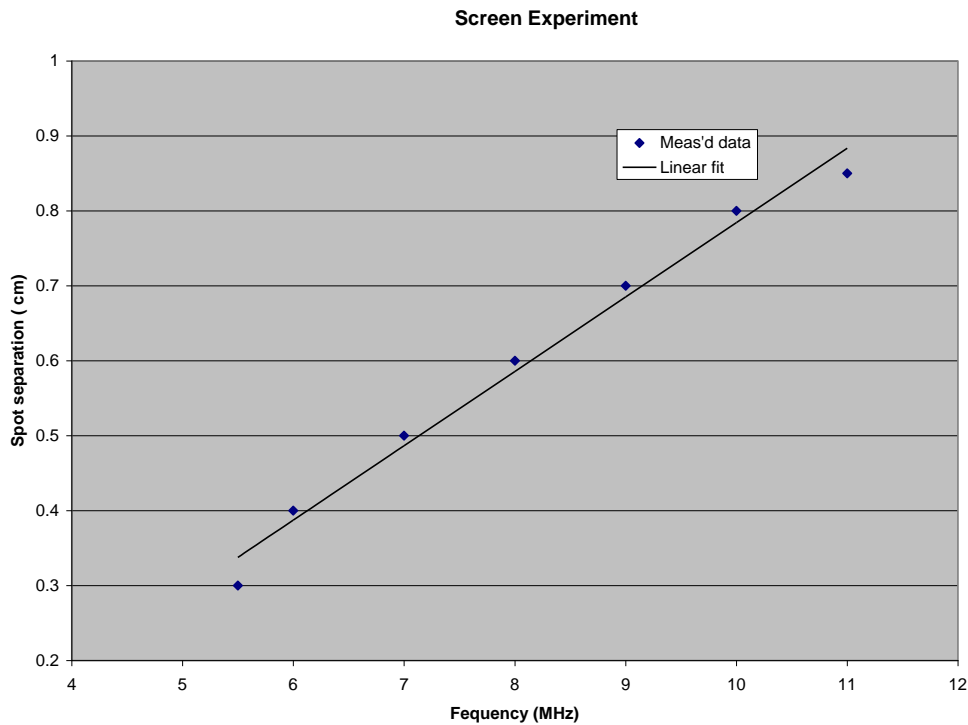


(b) As in (a) but for 10 slits. The diffraction peaks are now prominent but the interference pattern is just still visible.

One final comment: we are assuming here that the diffraction pattern is viewed at a large distance from the grating such that the wave fronts are effectively a plane. This is called Fraunhofer diffraction. If it is necessary to take account of the curvature of the wave fronts we have the much more complex situation of Fresnel diffraction, which we shall not treat here.

Before going on complete the interactive screen experiment. We'll discuss the results next.

If we return now to our experiment we see that it is a problem in diffraction.



Our results are shown above. Yours will be slightly different because you will have your own measurement errors. We've plotted the spot separation against oscillator frequency.

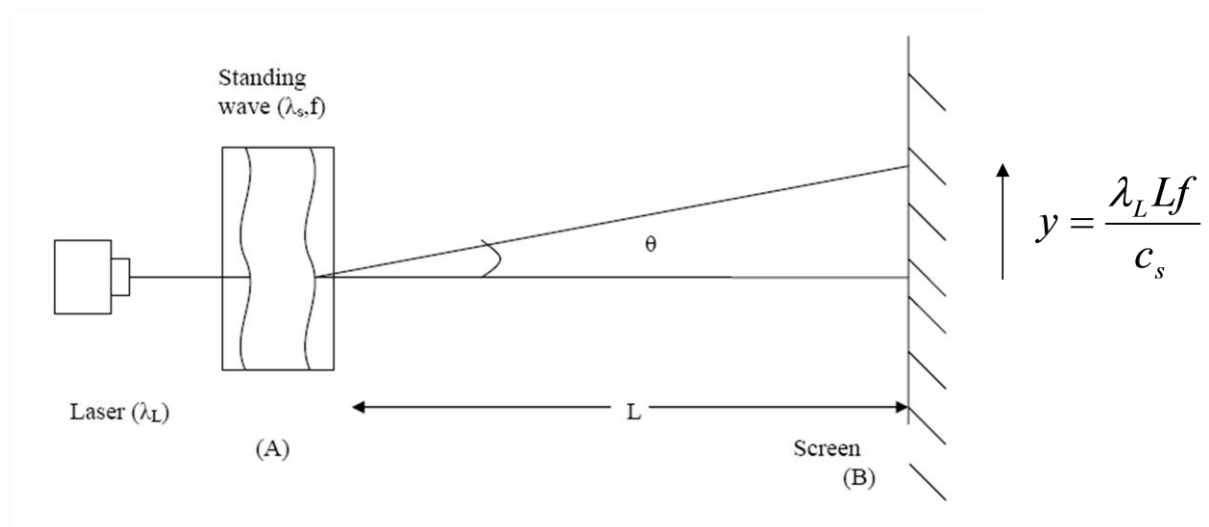
We assume that the water acts as a diffraction grating with spacing λ_s , the wavelength of the standing wave. From the grating equation in first order we derive the angle to the diffraction minimum, $\theta = \lambda_L / \lambda_s$. We already know $\lambda_s = c_s / f$. So we derive the straight line relation between the spot separation and the frequency. You should check that the slope you derive equals the one predicted by this equation.

Wavelength
of standing
wave

$d \sin \theta = m \lambda_L$

laser
wavelength

$$\sin \theta \approx \theta = \frac{\lambda_L}{\lambda_s} \text{ for } m=1 \quad \lambda_s = \frac{c_s}{f}$$



Summary

- Diffraction is the bending of waves on obstruction and the resultant interference pattern beyond the obstructed region
- The condition for a diffraction minimum for a slit of width a is $a \sin \theta = m \lambda$
- A diffraction grating produces a superposition of interference fringes and a diffraction envelope.

SAQs

1. Sunlight passing through a pinhole in a window blind produces a circular diffraction spot of diameter 1cm on a wall at a distance of 3 m. Given that the peak wavelength of sunlight is around 500nm give the diameter of the pinhole to 1 sf as the value of n in the expression $n \times 10^{-4}$ m.
2. As the width of the slits in a diffraction grating is reduced
 - a) The spacing of the diffraction minima increases
 - b) The spacing of the diffraction minima decreases
 - c) The spacing of the diffraction minima is unaffected
3. As the number of slits in a diffraction grating is increased their spacing is decreased and
 - a) The spacing of the interference maxima decreases
 - b) The spacing of the interference maxima increases
 - c) The spacing of the diffraction minima decreases

The answers appear on the following page

Answers

1. Correct: $n=2$
Incorrect feedback: if you use the radius instead of the diameter or the half angle instead of the full angle you will get an answer varying by factors of 2.

2. a) Correct: we are reducing the slit width a so the angle between diffraction minima is increased. Eventually the diffraction envelope disappears and we are left with an interference pattern from many narrow slits.
b) incorrect – since we are reducing the slit width a the angle between diffraction minima is increased
c) Incorrect – other things being equal, the width of the slits controls the diffraction pattern

3. a) Incorrect – we are reducing d so the angle between interference maxima is increased
b) correct – this is equivalent to reducing d so the spacing of the interference pattern is increased. The ratio ' d/a ' is reduced so the number of interference maxima between diffraction minima is reduced.
c) Incorrect – other things being equal the diffraction minima depend on the slit width not on the slit separation.

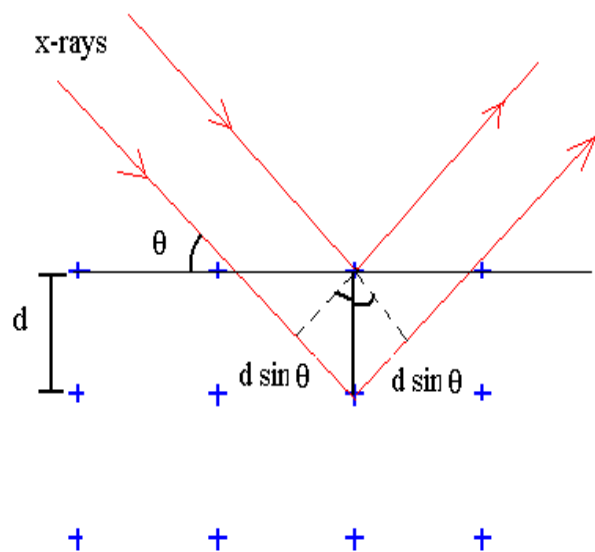
Additional Problems

Problem 1: Crystals as Diffraction Gratings

The problem is to derive the Bragg condition:

$$2d \sin \theta = m\lambda$$

for constructive interference of reflection from two successive layers of atoms in a crystal.



Answer:

The path difference between rays reflected from successive layers of the crystal is found from the two right angled triangles. It's just twice $d \sin \theta$. Constructive interference will occur if this is an integral number of wavelengths.

Total path length difference between rays

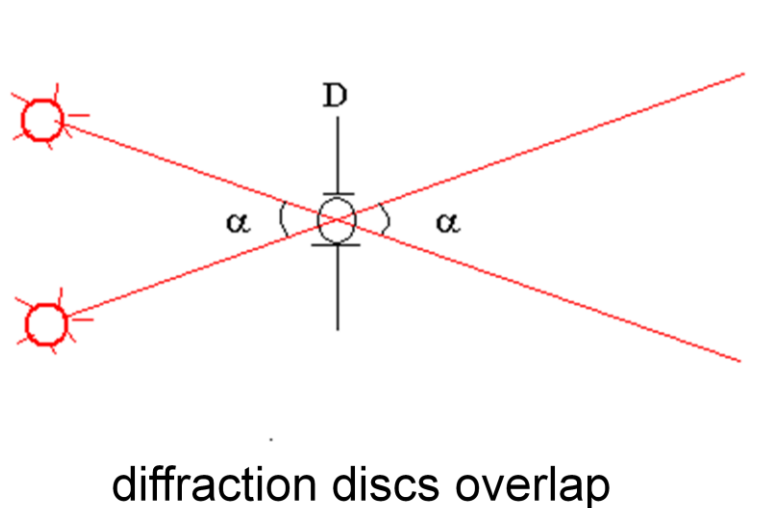
$$\Delta r = 2d \sin \theta$$

For diffraction maxima:

$$2d \sin \theta = m\lambda$$

Problem 2: Diffraction Resolution

Diffraction implies that close objects cannot be resolved because their Airy discs overlap.



The calculation of the radius of the disc is beyond the scope of this course, but the result is well known: it is that objects viewed in light of wavelength λ must be separated by an angle greater than $1.22\lambda/D$ if they are to be seen as separate images.

$$\sin \theta = \frac{1.22\lambda}{D}$$

For small angles $\sin \theta \sim \theta$

$$\theta = \frac{1.22\lambda}{D}$$

Use this information to estimate the angular resolution of the eye.

Answer: The eye will see two separate diffraction discs, and hence be able to resolve two images, if the angular separation exceeds $1.22\lambda/D$.

The resolution of the eye at say 400nm (blue light) is therefore given by $1.22 \times 4 \times 10^{-7} / 2 \times 10^{-3} = 2.4 \times 10^{-4}$ radians which is about 0.84 minutes of arc.

Overall Summary

- Waves that maintain a constant phase difference are said to be coherent
- The result of adding coherent waves depends on their phase difference δ : the combined amplitude is proportional to $\cos\delta/2$.
- The intensity depends on the square of the amplitude
- Coherent waves that traverse different optical paths can interfere constructively or destructively depending on the relative phase.
- A path difference Δx gives a phase difference $k \Delta x$
- The condition for an interference maximum for two thin slits is $d\sin\theta = m\lambda$
- Waves are diffracted by small obstacles
- The condition for a diffraction minimum for a slit of width a is $a\sin\theta = n\lambda$
- For a circular aperture of diameter D the diffraction disc has diameter $1.22\lambda/D$; this provides a limit to resolution of optical systems
- The Bragg condition for constructive interference in a crystal of lattice d is $2d\sin\theta = m\lambda$

Meta tags

Author: Naomi Banks.

Owner: University of Leicester

Title: Enhancing Physics Knowledge for Teaching – Wave Optics

Keywords: Interference; Diffraction; Superposition; Optics; Light; sfsoer; ukoer

Description: . In this session we shall basically be interested in the way visible light exhibits wavelike properties. These properties include the interference of light waves and the diffraction of light round obstacles.

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Language: English

Version: 1.0



Additional Information

This pack is the Version 1.0 release of the module. Additional information can be obtained by contacting the Centre for Interdisciplinary Science at the University of Leicester.

<http://www.le.ac.uk/iscience>

