Section 8: Electromagnetism

Section 7 included, among other things, an account of gravitational forces according to the classical Newtonian theory. This is the theory which says that any pair of objects will be attracted towards one another with forces proportional to the product of their masses and to the inverse square of the distance between them.

This section is devoted to forces which have many similarities with gravitation, namely electrical and magnetic forces. Electrical and magnetic forces are together known as "electromagnetic" forces because they are closely interconnected.

8.1 Electric Charges; Coulomb's Law

It has been known for a very long time that, when different materials are rubbed together, "electrical" phenomena may be produced. Sometimes objects will be made to stick together. Sometimes, they will repel one another.

This can be understood by imagining that "electricity" can be transferred from one material to another by friction. One thing gains electricity while the other loses it. In other words, one gains positive electrical charge and the other gains negative electrical charge.

Experiment shows that two positively-charged objects will repel one another. So will two negatively-charged objects. But if one object is positive and the other is negative, then there will be an attraction between them.

Like charges repel. Unlike charges attract.

Negative and positive signs were allocated to these charges quite arbitrarily in the early days of electricity. If you rub ebonite (a form of plastic, made from rubber) with fur, the ebonite acquires a negative charge. But if you rub glass with silk, the glass gains a positive charge. Nowadays this explained by the theory that matter consists of atoms with heavy positive nuclei and much lighter electrons. In many materials, the electrons are connected to the nuclei only very loosely and, under the right circumstances, can be made to move about. When two objects are given electric charges by friction, it just means that electrons are transferred from one to the other. Electrons are negatively charged (and atomic nuclei are positively charged) so an object which gains electrons becomes negatively charged, whereas an object which loses electrons becomes positively charged. In an uncharged object, the positive charges of the atomic nuclei are exactly balanced by the negative charges of the surrounding electrons. In a charged object, there is an imbalance, with either too many or too few electrons compared with the positive charges of the nuclei of the atoms.

In reality, therefore, a positive charge is a shortage of electrons and a negative charge is a surplus of them. And, since it is the electrons which are mobile, and not the atomic nuclei, a movement of charge in any ordinary material is actually a movement of electrons.

Different substances have different properties. In some, the electrons are tightly bound to the atomic nuclei or to the molecular structure and are difficult to move. In others, the electrons can drift about fairly freely. In a case where the electrons do not move, the substance is called an "insulator". In a case where the electrons are mobile, the substance is called a "conductor".

Electric charge is measured in units of coulombs (abbreviation C). The greater the charge, the greater the attraction or repulsion will be.

Incidentally, the unit of charge (coulomb) is related to the unit of electric current (ampere) because an ampere is a flow of one coulomb per second. For historical reasons the unit of current, not of charge, is taken to be a "base" unit in the SI system, alongside mass, length and time. So, a coulomb could be expressed as an "ampere-second" (A s) in base units.

Every electron has exactly the same electric charge. Electrons do not differ from one another, not even in the slightest. This fact is a deep mystery. The quantity of charge carried by an electron is -1.6×10^{-19} coulomb, the minus sign representing the fact that an electron's charge is regarded as negative. The magnitude of this charge is often represented by the letter e.

$$\boxed{e = 1.6 \times 10^{-19} \text{ coulomb}}$$

Electric charges arise (in almost all cases) because an object has lost or gained a number of electrons, so all electric charges are integral multiples of e. It can be said that electric charge is “quantised”, and that e is the “quantum” of electric charge.

Like charges repel. Unlike charges attract. The force of repulsion or attraction is present regardless of how far apart the two charged objects are, but the further apart they are the weaker the force will be. About two hundred years ago it was established that the force between two electrically charged objects follows the same mathematical rule as the gravitational force between two heavy objects: it is inversely proportional to the square of the distance between them.

So, if we have two objects with charges Q_1 and Q_2, and if they are a distance r apart, then the force pushing them apart obeys the equation

$$F \propto \frac{Q_1 Q_2}{r^2}$$

It is assumed here that the objects are very much smaller than the distance r, so that the shapes of the objects are not relevant. In other words, the objects are imagined to be pointlike. This equation is known as Coulomb's Law. Notice that both of the charges Q_1 and Q_2 "feel" the force F. It acts on Q_1 in one direction and also on Q_2 in the opposite direction. This is in accord with Newton's Third Law of Motion, which states that, for every force within a system, there is also an equal and opposite force.

Replacing the proportionality by an equality, we can write

$$F = \frac{1}{4\pi\varepsilon} \frac{Q_1 Q_2}{r^2}$$

where the factor $(4\pi\varepsilon)^{-1}$ is a constant analogous to the gravitational constant G which appears in Newton's Law of Gravitation. The quantity ε is known as the "permittivity". The above could be written as a vector equation:

$$\underline{\boldsymbol{F}}_1 \quad = \quad -\underline{\boldsymbol{F}}_2 \quad = \quad - \; \frac{1}{4\pi\varepsilon} \; \frac{Q_1 Q_2}{|r_{12}|^3} \quad \underline{\boldsymbol{r}}_{12}$$

following the convention of using bold-font symbols for vectors. Here, $\underline{F}_1$ is the force on Q_1, $\underline{F}_2$ is the force on Q_2, and $\underline{r}_{12}$ is the displacement of Q_2 from Q_1.

However, the electrical force law differs from Newton's Law of Gravitation in a very important way. While the gravitational constant G is a universal constant which always has the same value, the electrical permittivity ε varies depending on the material which is filling the space between the two charges Q_1 and Q_2.

If there is no material at all between the charges, just empty space, then the permittivity has a special value, ε_0, the "permittivity of free space". Since ε_0 does not depend on the properties of any material substance, just on the nature of the universe, it is a fundamental constant. In SI units it has a fixed value which is approximately equal to

$$\boxed{\varepsilon_0 \;=\; 8.854\ldots \times 10^{-12}\ \mathrm{kg^{-1}\,m^{-3}\,s^4\,A^2}}$$

Do not be intimidated by the complicated units. Here, the units of the permittivity are specified in terms of the "base units" of the SI system (mass, length, time and electric current). We will see later that the units of permittivity can be expressed a bit more simply than that.

Using the above value for ε_0, the force equation previously given will yield the correct result in newtons if the charges Q_1 and Q_2 are in coulombs and if the distance r is in metres.

Question 8a

Suppose that two small objects both have electric charges of 1 μC. If they are 1 metre apart, what is the electric force between them? (When you are asked any question of this kind, assume that the space between the objects is empty, i.e. is a vacuum, unless you are told otherwise.)

Solution: use

$$F = \frac{1}{4\pi\varepsilon}\frac{Q_1 Q_2}{r^2}$$

putting in Q1 = Q2 = 10^{-6}, r = 1, and $\varepsilon_0 = 8.854 \times 10^{-12}$ in the appropriate units. The result is a force of 9 mN (millinewtons).

Question 8b

A proton (i.e. the nucleus of an ordinary hydrogen atom) has a mass of 1.67×10^{-27} kilograms. A proton has the same electric charge as an electron, but is positive instead of negative. By what factor does the electrical force between two protons exceed the gravitational force between them? To save you looking it up: Newton's gravitational constant G is equal to $6.67 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$.

Solution: the formula for electrical force is

$$F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2}$$

and the formula for gravitational force is

$$F_g = G \cdot \frac{M_1 M_2}{r^2}$$

The charges Q_1 and Q_2 are both equal to the charge e given earlier, and the masses M_1 and M_2 are both equal to the proton mass (M, say). So, the ratio of the two forces, at any distance r, is

$$\frac{F_e}{F_g} = \frac{e^2}{4\pi\varepsilon_0 G M^2}$$

Putting in the numbers, this gives a factor of just over 12 x 10^{27}. It is because this is such an enormous ratio that gravitational forces can be entirely neglected in the context of atomic and nuclear forces.

8.2 Electric Fields

Equations have been given above for the force between two electrically-charged pointlike objects. But what happens if there are more than two objects, or if the electric charge is spread out over a region of space? To be able to think about electric charges and electric forces in a general way, we need the concept of the electric "field".

At every point in space, the electric field gives the magnitude and direction of the electric force that would act on a small positive charge located at that point, if such a charge were present there. To be precise: if a charge q were to be present on a pointlike particle at the location whose coordinates (x, y, z) form the vector $\underline{r}$, and if the particle would then be acted on by an electric force $\underline{F}$, then the ratio $\underline{F}$/q is the "electric field" at $\underline{r}$ and is conventionally denoted by the letter $\underline{E}$. $\underline{E}$ is a vector because $\underline{F}$ is a vector. The electric field $\underline{E}(\underline{r})$ is a vector-valued function of a vector variable, so it is mathematically rather a sophisticated thing. Moreover, it is defined in terms of something that does not exist – the imaginary pointlike positively-charged particle at the location $\underline{r}$.

Sometimes the electric field is described in terms of a "test particle of unit charge" imagined to be placed at the point in question: the electric field is defined as the electrical force which would act on a test particle of unit charge at the relevant point. An electric field has the units of newtons per coulomb.

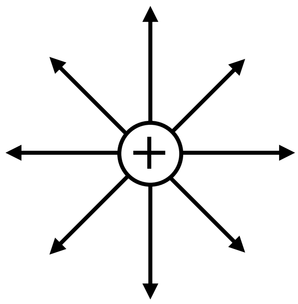
An electric field surrounds any charged object. From Coulomb's Law, as discussed above, a charge Q will exert a force $\underline{F}$ on another charge q, at a separation $\underline{r}$ from it, with

$$\underline{F} = \frac{1}{4\pi\varepsilon} \frac{Qq}{r^3} \underline{r}$$

and since the electric field at that point is $\underline{F}$/q, we can write

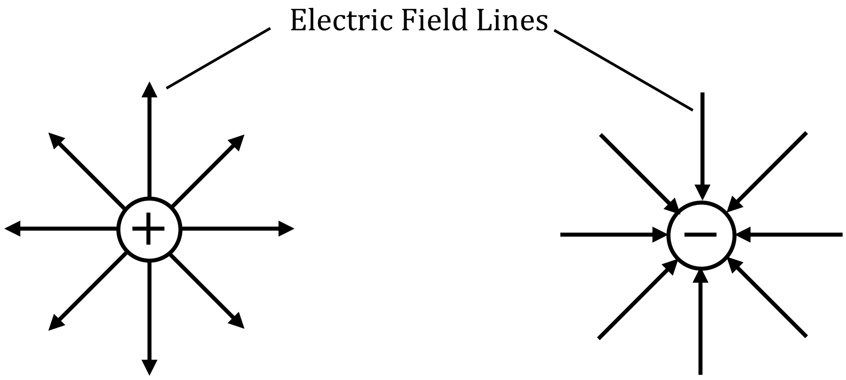
$$\underline{E} = \frac{Q}{4\pi\varepsilon} \frac{\underline{r}}{r^3}$$

This expresses, mathematically, the fact that a small charged object by itself will produce all around it a radial electric field (in the direction of the separation $\underline{r}$) whose magnitude is proportional to the charge Q on the object and to the inverse square of the distance r. This electric field could be illustrated by a diagram like this:



The lines follow the direction of the field at every point. Field lines always start from where there is an electric charge, which is in the centre of the diagram in the above example. The diagram above should be imagined in three dimensions with the field lines spreading out and becoming increasingly sparse as we go away from the central charge and the magnitude of the electric field falls away.

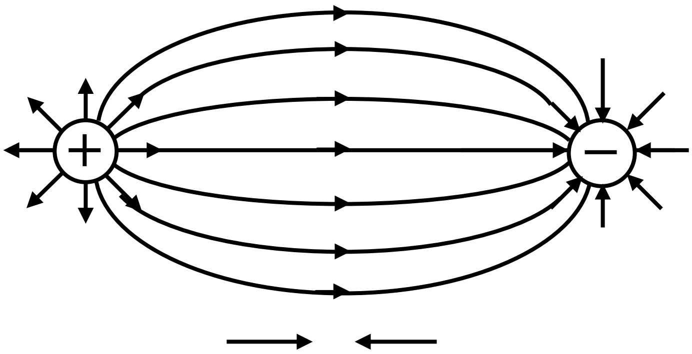
If we had two charges side by side, one positive and the other negative, than we might have a situation like this:



Electric field produced by a positive charge

Electric field produced by a negative charge

but the combined result would actually be something like this:



What happens is that, at every point, the electric field is the vector sum of the field due to the positive charge and the field due to the negative charge. Field lines flow from the positive charge to the negative charge. You can imagine the field lines as being almost like stretched elastic, pulling the positive and negative charges together.

It is important to understand the role of the permittivity ε. If there is just empty space between the charge Q and the point $\underline{r}$, then the permittivity is just that of the vacuum, ε_0, and we would have

$$\underline{E} = \frac{Q}{4\pi\varepsilon_0}\,\frac{\underline{r}}{r^3}$$

However, if the intervening space is filled with some other medium, then the permittivity ε will have some other value greater than ε_0. For example, air has a permittivity ε which is greater than ε_0 by a factor of 1.0006. The ratio $\varepsilon/\varepsilon_0$ is known as the "relative permittivity". The relative permittivity of air is 1.0006. All materials have permittivities greater than that of the vacuum, so their relative permittivities are above 1. Some materials have relative permittivities which are considerably greater than 1: for example, glass has a relative permittivity of about 6 and distilled water has a relative permittivity of 81. Relative permittivity is often represented by the letter κ (Greek lower-case kappa).

What is the significance of the relative permittivity? Looking at the above equation, it can be seen that a high permittivity will reduce the electric field. In other words, a high-permittivity material has the effect of partially screening the electric field. Exactly how this happens depends on the atomic and molecular structure of the material, and need not concern us here. What needs to be understood is simply that materials are not, so to speak, completely porous to electric fields, and in a high-permittivity medium the power of an electric charge to produce an electric field will be proportionately reduced.

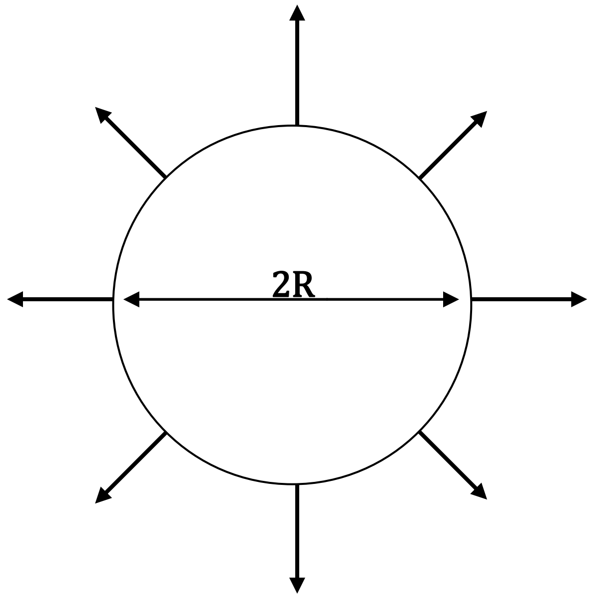
In this respect, electrically conducting materials are in a class of their own. A conductor, in effect, is a material with an extremely high permittivity. Consequently, the power of an electric charge to create an electric field is quenched completely within a conductor. Why? One way of looking at it is to say that a conductor is a material in which electrons are highly mobile and therefore electric currents will pass with very little resistance. So, if a particular distribution of charge creates an electric field within a conductor, a current will be made to flow, with the result that the charges will be redistributed until they balance out and the electric field therefore eliminated.

If an isolated object made of a conducting material carries a net electric charge, then the self-repulsion of the charge will quickly spread it out over the surface of the material.

Incidentally, because there can be no electric field inside a conductor, any equipment which is contained within a conducting surface will be shielded from any extraneous electric fields. Or, to put it in another way, the super-high permittivity of a conductor will prevent any electric field from penetrating to equipment which is protected by a conducting box. This is the principle of the "Faraday cage" used to shield sensitive electrical equipment. It is not always necessary for the Faraday cage to be a completely closed: a mesh or cage made from conducting wires will give considerable protection.

Charge distributed over a surface

A configuration which it is important to understand is a uniform distribution of charge over a flat surface. What sort of electric field is produced by such a distribution? As a way of approaching this problem, imagine a conducting sphere of radius R, as shown below, with a positive charge Q is distributed evenly over the surface of the sphere.



The surface area of the sphere is $4\pi R^2$, and so the charge density (per unit area) over its surface is given by

$$\sigma = \frac{Q}{4\pi R^2}$$

Now think about the electric field at a point outside the sphere. At a very large distance (r, say, $\gg$ R) away from the sphere, the sphere will seem very tiny, and its electric field cannot be significantly different from that of a point charge. The field would therefore be

$$E = \frac{Q}{4\pi\varepsilon r^2}$$

As we go towards to the surface of the sphere, $r \rightarrow R$, the formula will not change. As long as we are outside the sphere, the field lines must look the same and there must be the same number of them whether the charge is smeared over the surface of the sphere or concentrated at its centre. In fact the situation is analogous to that for the force of gravity, where any spherical object behaves as if its mass were concentrated at its centre. The magnitude of the field very close to the surface of the sphere must therefore be

$$E = \frac{Q}{4\pi\varepsilon R^2}$$

So, using the equation above for the charge density σ, it is clear that the electric field just above the surface of the sphere must be simply

$$E = \frac{\sigma}{\varepsilon}$$

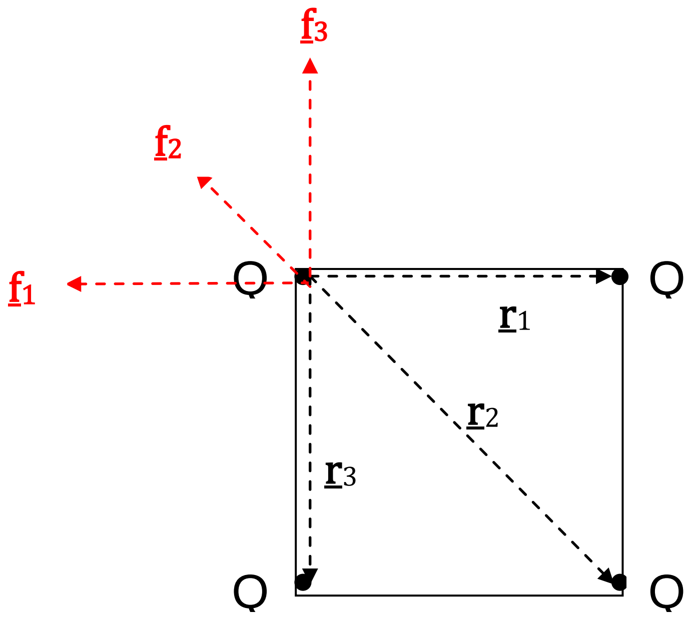
Now, there can be no difference between a spherical surface with a relatively very low curvature (i.e. a sphere as seen from a point very close to it) and a plane surface. We conclude that the electric field just outside any flat distribution of charge σ (in coulombs per square metre) will be σ/ε (newtons per coulomb).

But a word of warning is necessary: we have argued here that the electric field due to a plane distribution of charge (σ C m^{-2}) has a magnitude equal to σ/ε. But this is in circumstances where the electric field is produced only on one side of the plane. As we will see later on, the magnitude of the electric field is only half of this (σ/2ε) if the field is emitted on both sides of the plane.

Question 8c

Four small electric charges, each of Q = 5 μC, are at the corners of a square whose sides are 0.1 metres in length. What is the magnitude of the electrostatic force felt by each of the charges?

Solution: first we draw a diagram...



The force felt by the charge at the top left-hand corner is the vector sum of the forces ($\underline{f}_1$, $\underline{f}_2$ and $\underline{f}_3$, say) which it feels from the three other charges:

$$\underline{F} \;=\; \underline{f}_1 + \underline{f}_2 + \underline{f}_3$$

and each of those follows the inverse-square law:

$$\underline{F} \;=\; -\;\frac{Q^2}{4\pi\varepsilon_0}\;.\;\left[\frac{\underline{r}_1}{{r_1}^3} \;+\; \frac{\underline{r}_2}{{r_2}^3} \;+\; \frac{\underline{r}_3}{{r_3}^3}\right]$$

Using the permittivity of empty space:

$$\varepsilon_0 \;=\; 8.854 \;\times\; 10^{-12}\;\; kg^{-1}\, m^{-3}\, s^4\, A^2$$

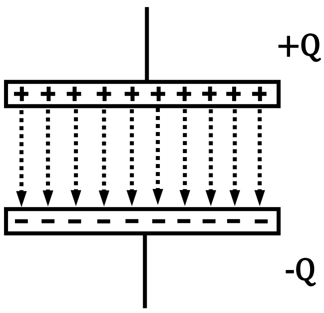
we find that the net force is in the opposite direction to $\underline{r}_2$ and has magnitude

$$\frac{\left(5 \times 10^{-6}\right)^2}{4\pi \times 8.854 \times 10^{-12}} \;.\; \frac{1}{(0.1)^2} \;.\; \left[\frac{1}{\sqrt{2}} \;+\; \frac{1}{2} \;+\; \frac{1}{\sqrt{2}}\right]$$

$$= \;\; 43 \text{ N}$$

8.2 Capacitance

We are now equipped to consider the extremely important example of two similar parallel flat plates:



In this diagram, electric field lines are shown going from the upper plate (which carries a total charge Q) down to the lower plate (which carries a charge -Q. As an element of an electrical circuit, this is what is called a "capacitor".

As long as the size of each plate is much greater than the separation between them the diagram above will be a pretty accurate representation of how the electric field goes. It must be perpendicular to the plates, and must be the same across the area of each plate, since the electric charge will spread itself out uniformly across the surface of each plate. (There will be some "fringing" close to the edges of the capacitor, with the field bending out sideways a bit, but this effect will be negligible if the distance between the plates is relatively small.) Since the electric field lines are parallel to one another, they do not diverge and therefore the electric field does not vary vertically between the plates.

Suppose that the area of each plate is A and that the distance between them is d. Then, the areal charge density on each plate has magnitude

$$\sigma = Q/A$$

and so the electric field between the plates has a magnitude

$$E = \frac{\sigma}{\varepsilon} = \frac{Q}{A\varepsilon}$$

Now, imagine the change in energy involved when electric charge builds up on a capacitor. When charge is transferred from one plate of the capacitor to the other, it is as if charge is moved against the force produced by the electric field E. Work will have to be done on the charge, and it will acquire potential energy just as if it had been rolled up a steep hill. This sort of potential energy, when expressed in joules per coulomb, is often termed "potential difference". Actually, we don't have to use the phrase "joules per coulomb" because there is a special name for this unit: the volt (abbreviation V). There is an electrical potential difference, in other words a voltage, between any two points in space, and it is equal to the energy that would have to be released or absorbed in order to move 1 coulomb of charge from one of the points to the other.

So, what is the electric potential energy difference between the plates of a capacitor? The potential difference must be equal to the electric force on a unit of charge between the plates (which, by definition, is the electric field E) multiplied by the distance d that would be travelled between the two plates.

This is because for any kind of force:

$$\boxed{\text{energy} = \text{force} \times \text{distance}}$$

And consequently

$$V = Ed$$

(Confusingly, V is not only the abbreviation for the unit of voltage, the "volt", but it is also the letter conventionally used to represent a voltage when doing algebra.)

Eliminating E between the last two equations, we can write

$$Q = \frac{A\varepsilon}{d} . V$$

To look at it from another point of view: if the voltage V is imposed on the capacitor by an external electrical circuit connected to the two plates (using, say, a battery) then the voltage will cause the capacitor to be "charged up" with positive and negative charges $\pm$Q which will be proportional to V according to the above equation. For any particular capacitor, A, ε and d are constants, and if we define the "capacitance" of the capacitor to be the ratio

$$C = \frac{Q}{V}$$

then it follows that

$$C = \frac{A\varepsilon}{d}$$

(Notice that we talk colloquially about the charge Q on a capacitor although in fact the capacitor has two charges, +Q and -Q, and the total charge on the whole capacitor is always therefore zero.)
From the equations given earlier, the capacitance of a capacitor has the units of coulombs per volt, which is equivalent to $\mathrm{kg}^{-1}\ \mathrm{m}^{-2}\ \mathrm{s}^4\ \mathrm{A}^2$ in SI base units. The unit of capacitance is given a name of its own, the farad (abbreviation F) in memory of the physicist Michael Faraday. In terms of farads, permittivity has units of farads per meter ($\mathrm{F\ m^{-1}}$). Also, notice that electric field strength *"E") can be expressed as "volts per metre" ($\mathrm{V\ m^{-1}}$) as well as newtons per coulomb ($\mathrm{N\ C^{-1}}$) - they are equivalent ways of expressing the same base units of $\mathrm{kg\ m\ s^{-3}\ A^{-1}}$.

Question 8d

Two parallel plates 3 mm apart gain a charge of 35 nC when connected to a 150V DC voltage supply. The effective cross-sectional-area of each plate is 144 x 10^{-4} m^2. Calculate the electric field strength between the plates.

Solution: *Electric field strength* $E = V/d = 150/(3 \times 10^{-3})$

$$= 50\,000 \text{ V m}^{-1} = 50\,000 \text{ N C}^{--1}$$

It does not matter whether we give the units as volts per metre or newtons per coulomb. Both are correct. It's just a shame that the unit of electric field strength doesn't have a special name of its own!

Question 8e

Two parallel metal plates of dimensions 0.25 m x 0.35 m are spaced 4 mm apart. The plates receive a charge of 250 nC from a 220 V supply. Calculate the electric field strength.

Answer: 55 kV m^{-1}

Question 8f

A charge of 0.5 μC is carried on two rectangular plates of dimensions 60 mm x 80 mm. The distance between the plates is 1 mm and a potential difference of 500 V is connected across the plates. Calculate the capacitance, the permittivity and the relative permittivity.

Solution: The capacitance can be found from the charge and voltage with the formula $C = Q/V$. *So,* $C = (0.5 \times 10^{-6})/500 = 10^{-9}$ *F or 1 nF.*

But the capacitance $C = A\varepsilon/d$, *so* $\varepsilon = Cd/A = 10^{-9} \times 10^{-3} / (4.8 \times 10^{-3}) = 0.21 \times 10^{-9}$ F m^{-1}. *The relative permittivity* $\varepsilon/\varepsilon_0 = (0.21 \times 10^{-9})/(8.85 \times 10^{-12}) = 23.73$.

The energy in an electric field

Imagine a parallel-plate capacitor being slowly charged up. At first, there are no electric charges on the plates, but they build up to having charges of +Q and –Q. Charge is steadily transferred from one plate to the other. Because there will be a potential difference between the two plates, work is done as the charge is transferred. As far as the capacitor is concerned, it doesn't matter what sort of external circuitry is involved in this process, as long as the net effect is to transfer charge from one plate to the other.

How much energy is involved in this process? When the charge on the capacitor is q, then the voltage V between the plates must be q/C, where C is the capacitance. If that is the voltage, then the transfer of an extra bit of charge Δq must require an energy input of $V\Delta q = q\Delta q/C$.

This is analogous to what happens when we put energy into stretching a spring. Hooke's Law says that there is a force $F = kx$ in a spring which has been extended by a distance x. So, when we increase x by another bit Δx, there is an extra energy input (force multiplied by distance) equal to $kx\Delta x$. When the spring is extended to give a total extension of X, the energy stored in the spring is $½kX^2$. This can be shown either by using calculus (integrating $kx\Delta x$) or by drawing a graph of the force as a function of x and using the fact that the energy must be the area under the graph. In the case of the capacitor, the energy is $½Q^2/C$. Since $V=Q/C$, this is equal to $½CV^2$, which is the usual formula for expressing the energy stored in a capacitor.

So, a capacitor not only stores positive and negative electric charges, it also stores energy. But where, exactly, is this mysterious energy? The answer is that energy is inherent in an electric field, and the energy stored in a capacitor, $½CV^2$, is the energy of the electric field between the two plates.

Now, as we have seen, the capacitance C is $A\varepsilon/d$ and the voltage V is Ed. Using these formulae to replace C and V in the expression $½CV^2$, we can see that the energy stored in the capacitor is $½Ad\varepsilon E^2$. But Ad is the volume of the space between the two plates. Since the energy is stored in that volume, the energy density (in joules per cubic meter) must be is $½Ad\varepsilon E^2/Ad$ which is just $½\varepsilon E^2$.

So, the example of the parallel-plate capacitor has led us to the conclusion that an electric field E must have, associated with it, an energy density given by

$$\boxed{Electric\ field\ energy\ density\ =\ ½\varepsilon E^2}$$

Question 8g

The plates of a parallel plate capacitor each have an area 25×10^{-3} m^2 and are separated by an air gap of 5mm. The electric field strength between the plates is 70 kV m^{-1}. Calculate the capacitance of the capacitor and the energy stored by it.

Answers: 44.25×10^{-12} *F and* 2.71×10^{-6} *J*

Question 8h

Two parallel metal plates of area 0.8 m^2 and separated by an air gap of thickness 1 mm have a voltage of 200 V connected across the plates.

Calculate (a) the capacitance, (b) the charge, and (c) the new value of capacitance and charge if the distance between the plates is halved. (d) If the supply across the capacitor in part (c) above is now disconnected and the distance between the plates is then returned to its original value, what will be the new voltage difference across the plates?

Solution: let the initial values of the capacitance, charge and voltage be C_1, Q_1 and V_1, and let the initial separation of the plates be d_1.

(a) *For air it is reasonably accurate to take $\varepsilon = \varepsilon_0 = 8.85 \times 10^{-12}$ F/m. The area $A = 0.8$ m^2, $d_1 = 10^{-3}$ m, and $V_1 = 200$ V. So, the capacitance $C_1 = \varepsilon_0 A/d_1 = 8.85 \times 10^{-12} \times 0.8 / 10^{-3}$ **=** 7.08×10^{-9} F*

(b) *Charge $Q_1 = C_1V_1 = 7.08 \times 10^{-9} \times 200$ **=** 1.416 μC*

(c) *If the distance between the plates is halved, the capacitance will be doubled since capacitance is inversely proportional to the distance between the plates. So, the new capacitance $C_2 = 2 \times C_1$ **=** 14.16×10^{-9} F or 14.16 nF. Also, the charge $Q_2 = C_2V_1$, so if the capacitance is doubled and the voltage remains constant the charge must also double. So, the new value of the charge $Q_2 = 2 \times Q_1$ **=** 2.832 μC.*

(d) *If the supply across the capacitor in part (c) above is now disconnected and the distance between the plates is then returned to its original value the capacitance will also return to its original value C_1 of 7.08 nF. However, with the supply disconnected the capacitor cannot discharge, therefore the charge will remain at its new value (Q2) of 2.832 μC. Therefore, the new voltage V2 = Q2/C1 = $(2.832 \times 10^{-6})/(7.08 \times 10^{-9}) = 400$ V. So, the voltage has increased. This is possible because work has been done on the capacitor by moving the plates apart against the force of attraction between the plates. More energy has been put into the capacitor and., since voltage is joules per coulomb, the voltage must increase.*

Question 8i

A 5 nF parallel-plate capacitor, with empty space between the plates, is charged by a set of batteries to 160 volts. It is then disconnected from the batteries. Then, it is submerged in distilled water. The relative permittivity (i.e. $\varepsilon/\varepsilon_0$, or κ) of distilled water is 80.

(a) What is the capacitance of the now-water-filled capacitor?

(b) What is now the voltage across the capacitor?

(c) How much energy was stored in the capacitor before immersion?

(d) How much energy is stored in the capacitor after its immersion in the water?

Solutions:

(a) Capacitance $C = A\varepsilon/d$, where A is the area of the plates and d is their distance apart. So, if the permittivity goes from ε_0 to a value 80 times greater, then C is multiplied by 80. The new capacitance is therefore $80 \times 5 = 400$ nF.

(b) When the batteries are disconnected, the charges on the capacitor's plates remain unchanged. With no electrical circuit connecting them, there is nowhere for the charges to flow. However, the voltage between the plates may change if the configuration of the capacitor changes in any way.

The voltage across a capacitor depends on the capacitance C and the charge Q according to the equation $V = Q/C$. So, if the capacitance C is multiplied by 80 while Q remains the same, the voltage V must fall by a factor of 80. The voltage across the submerged capacitor is therefore $160/80 = 2$ V.

(c) The energy stored in by a capacitor is given by the formula $½CV^2$ (or $½Q^2/C$). Out of water, using $C = 5$ nF and $V = 160$ V, this gives 64 µJ.

(d) In water, using $C = 400$ nF and $V = 2$ V, the energy stored ($½CV^2$) comes to 0.8 µJ.

The energy stored in the capacitor has been reduced by a factor of 80 as a result of its immersion in water. Where did the energy go?

Question 8j

A parallel-plate capacitor of capacitance C is connected via an electrical circuit to a battery which gives it a charge Q. The capacitor is then disconnected from the circuit.

(a) In terms of C and Q, how much energy (U, say) is stored in the capacitor?

(b) The separation between the plates is now doubled. How does the energy in the capacitor change?

Solutions:

(a) *This just requires the standard formula $U = ½Q^2/C$. For the answer to this first part of the question it does not actually matter whether the electrical circuit is connected or not.*

(b) *Remember (or look up!) the fact that $C = A\varepsilon/d$. This means that if the plate separation d is doubled, the capacitance C is halved.*

But the energy in the capacitor is $U = ½Q^2/C$, and the charge Q is unchanged (because, with the circuit disconnected, the charges on the plates cannot flow anywhere and have to stay where they are). So, with the capacitance C being halved, the energy U is doubled.

If the energy U in the capacitor has been doubled as a result of the plate separation being doubled, where did that extra energy come from? The answer is that the mechanical work done in pulling the plates apart is converted into the extra electrical energy in the capacitor.

Now let's look at that problem again, but with a crucial change: this time the capacitor will not be disconnected from the circuit! So, the question is:

Question 8k

A parallel-plate capacitor of capacitance C is connected via an electrical circuit to a battery of voltage V which gives it a charge Q.

(a) How much energy is stored in the capacitor?

(b) The separation between the plates is now doubled. How does the energy in the capacitor change?

Solutions:

(a) *It is still true to say that the energy U in the capacitor is $½Q^2/C$.*

(b) *However, as the plates are pulled apart both Q and C will change. So, it is best to eliminate Q (using $C = Q/V$) to give $U = ½CV^2$. Then, use $C = A\varepsilon/d$ to reach*

$$U = A\varepsilon V^2/2d$$

With the energy in the capacitor being expressed in this form, it is obvious that by doubling d we will halve U.

So, when you double the separation of the plates of a charged capacitor, you will double the energy stored if the capacitor is disconnected (constant Q) but you will halve the energy stored if the capacitor stays connected to the battery (constant V)!

But now we have another little puzzle. In this case, with the capacitor left connected to the battery, increasing the plate separation decreases the energy in the capacitor. But mechanical work is still being done when we pull the plates apart. Where is that mechanical work going? And where is the electrical energy from the capacitor going? The answer is: into the battery. As we pull the plates apart, we charge the battery up!

The force between the plates of a capacitor

Consider now a parallel-plate capacitor which has its plates separated by a distance d. If its capacitance is C, and there is a charge Q on the capacitor, what is the force between the plates?

To answer this question, we must first understand the following:

> If a system consists of a number of electric charges at points A, B, C, D, E, F, G, H etc., then the electrostatic force felt by one of them (say, the one at A) is given by the formula $Q_A\underline{E}_A$ where Q_A is the charge at A and $\underline{E}_A$ is the electric field which would be produced at point A by the charges at B, C, D etc. *if the charge at A were not present*.

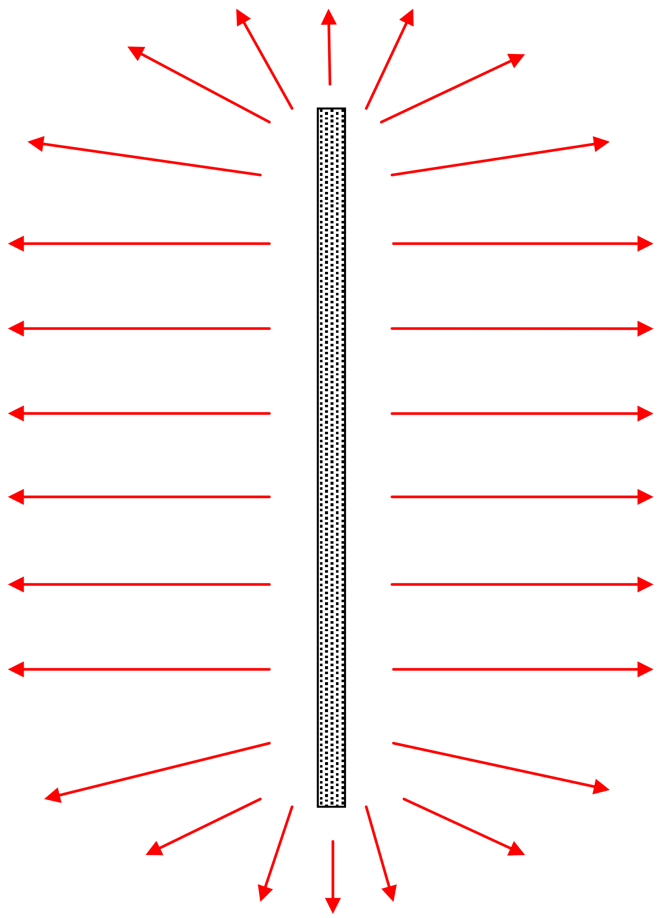
The phrase "if the charge at A were not present" is crucial. (In fact, if the charge at A were included then the electric field at A would be impossible to define because of the mathematical infinity that you get from the inverse square law (field $\propto r^{-2}$) when $r \rightarrow 0$!)

In the case of the parallel-plate capacitor,

> the force on one plate is the charge which it carries multiplied by the electric field which would be there if only the other plate existed.

This is emphasised because it is often a source of confusion or puzzlement among students.

So, what would be the electric field if there were only a single plate? The electric field from a single positively charged plate would like this:



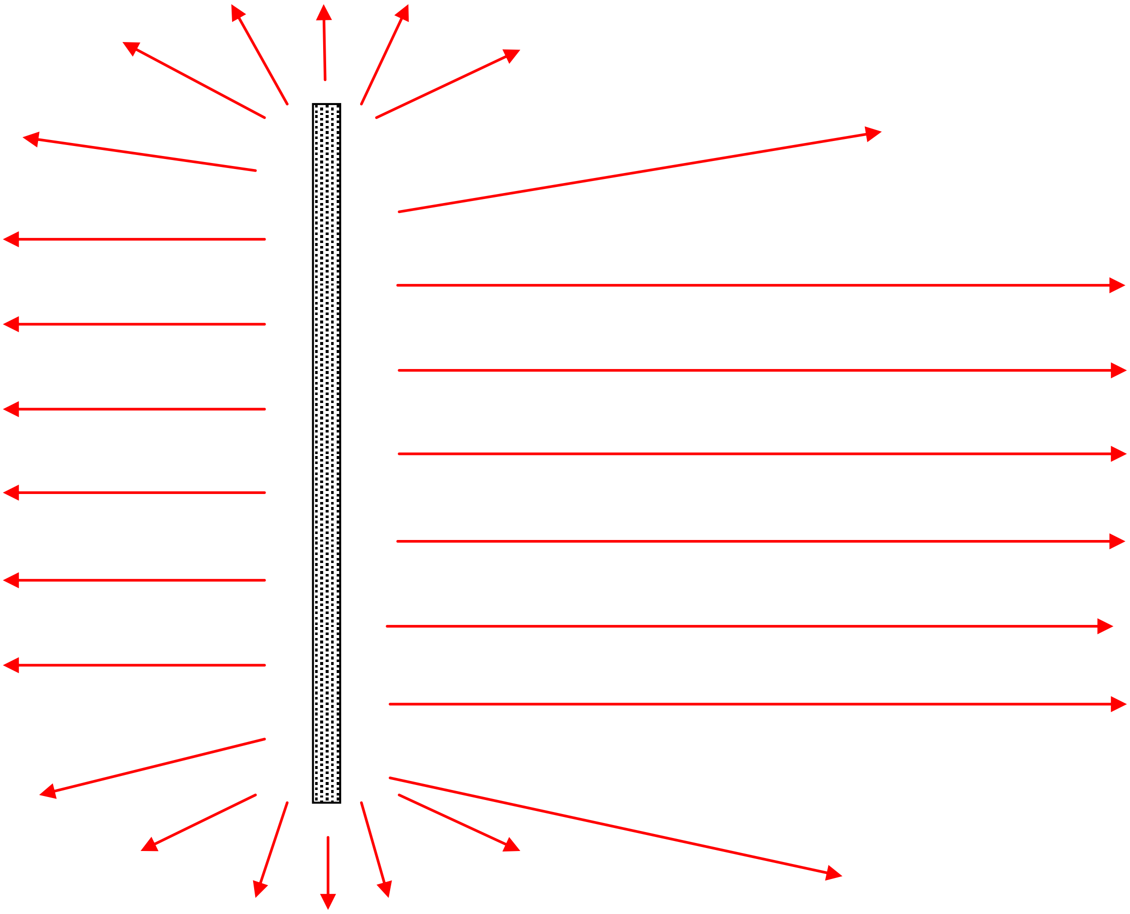
and the magnitude of the field (close to the plate and not near the edges) will be $\sigma/2\varepsilon$ (not σ/ε, because the field goes out on both sides of the plate). So, if a second plate (carrying charge -Q) is put beside that, the force on the second plate should be given by

$$\text{force} \quad = \quad \text{field x charge} \quad = \quad \sigma/2\varepsilon \quad \times \quad (-Q)$$

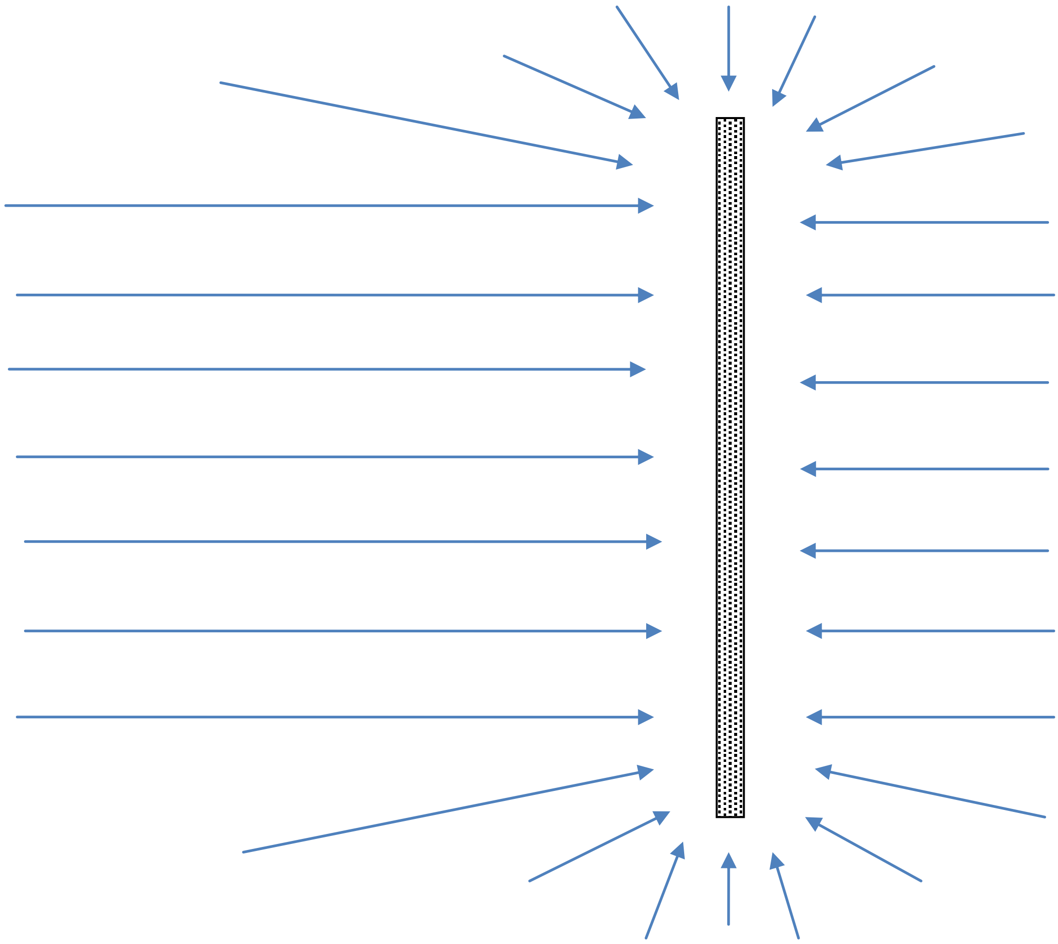
but if the area of each plate is A, the areal charge density is $\sigma = Q/A$, so we arrive at

$$\text{force} \quad = \quad - \tfrac{1}{2} \, . \, Q^2/A\varepsilon$$

the negative sign indicating repulsion. This answers the question as posed; but to go a little further let's redraw the figure above with the positively-charged plate on the left:



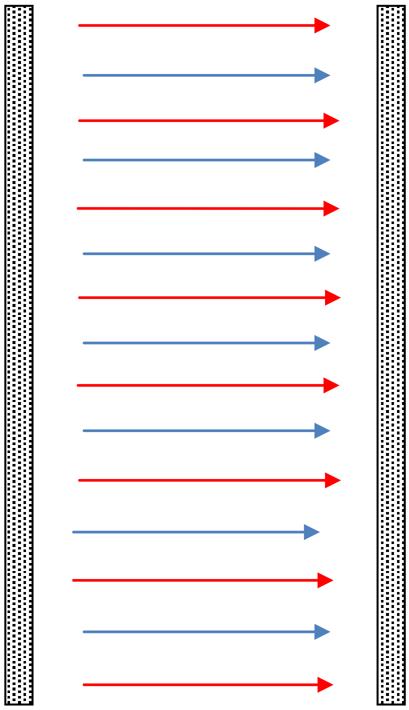
the reason being that we are going to draw in the second plate on the right. The second plate is negatively charged, and the field due to it is illustrated in the next diagram:



Now, imagine the two diagrams above being superimposed on one another.

Whenever two charged objects, or sets of objects, are brought together, the resultant electric field is simply the vector sum of the electric fields that the objects produced separately.

The superposition of the two diagrams above just looks like the next diagram:



Why does it look like this? The reason is that the outside the capacitor, the fields due to the two plates (the red and blue fields) cancel each other out, because the arrows there are going in opposite directions. But inside the capacitor, between the plates, the arrows are going in the same direction and the fields therefore reinforce one another. So, although the field due to one plate is equal in magnitude to $\sigma/2\varepsilon$, the field between two plates is twice this, i.e. σ/ε.

It was mentioned above that the force on each plate is given by the formula $\frac{1}{2} \cdot Q^2/A\varepsilon$. Now, suppose we are talking about a capacitor which is not connected to any external electrical circuit. That being the case, the charge Q is fixed and therefore the force between the plates is fixed, even if the distance between the plates is increased or decreased.

Now, imagine starting from a situation where the plates are practically touching one another. There would then be no electrostatic field energy in the capacitor. Now imagine that we gradually increase the separation, doing work against the constant force $\frac{1}{2} \cdot Q^2/A\varepsilon$ until there is a distance d between the plates. The mechanical work done must be equal to the force multiplied by the distance d. This is equal to

$$\frac{1}{2} \cdot (Q^2/A\varepsilon) \cdot d \quad = \quad \frac{1}{2} \cdot Q^2/C$$

where we have used the fact that the capacitance (when the separation has reached d) is given by $C = A\varepsilon/d$. This can also be expressed in terms of the final voltage V between the plates, using $C = Q/V$, to give the energy in the capacitor as $\frac{1}{2}CV^2$. This is the same as the formula that was obtained earlier by a different line of argument.

8.3 Magnetic fields

At the simplest level, for systems in which there is no movement of any kind, it is possible to understand magnetism in much the same way as electricity. "Electrostatics" is the science of the electric forces among stationary electric charges. Similarly, "magnetostatics" is the science of stationary magnetic fields. The difference between them is that electric charges actually exist, whereas magnetic "charges" do not. Magnetic fields are produced by magnets, but a magnet always seems to contain not one but two magnetic "charges" which are called "poles". The phrase "seems to" is crucial, because magnetic poles are, basically, illusions. Nevertheless, the concept of magnetic poles allows us to describe the behaviour of magnets and to calculate magnetic forces.

For historical reasons, magnetic poles are usually described as being "north" or "south" instead of positive or negative. A magnet, in its simplest form, consists of a bar made from a suitable magnetic material, and it will seem that a north pole is located near one end of the bar and a south pole near the other. The poles having equal and opposite strengths, the bar as a whole will have no net magnetic "charge". There are various magnetic materials from which such a magnet can be made: iron, nickel, cobalt and a number of other metals are suitable. Such materials are called "ferromagnetic" if, like iron, they can be magnetised so as to produce powerful permanent magnets. The phrase "permanent magnet" is used because, once the bar has been magnetised, it can retain its magnetisation more or less indefinitely: by contrast, an "electromagnet" is a different kind of magnet which is sustained by the flow of an electric current and will cease to be a magnet as soon as the current is switched off. Exactly how a magnetic field can be sustained by an electric current will be considered at a later stage.

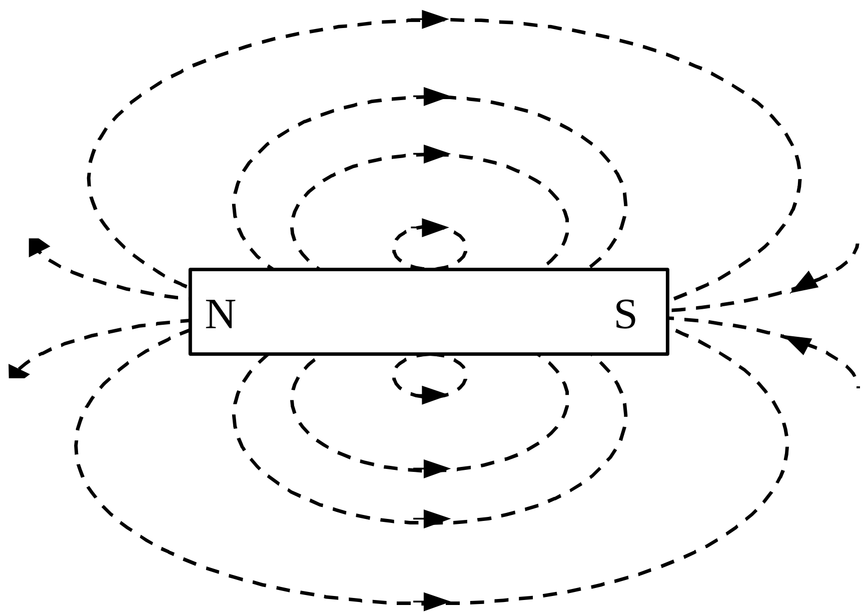
Historically, our understanding of magnetism developed in the context of the discovery that the Earth itself is a giant magnet. Crudely speaking, the Earth behaves as if it has a magnetic pole beneath the geographical North Pole and an opposite magnetic pole beneath the South Pole. Actually, if the geographical North and South poles are taken to be the points on the surface through which the Earth's axis of rotation passes, then the Earth's magnetic poles are not precisely beneath the geographical poles but are beneath points on the surface which are some hundreds of miles away. Moreover, the magnetic poles appear to be a long way beneath the surface, near the boundary between the Earth's core and its mantle.

Opposite magnetic poles (i.e. a north pole and a south pole) are attracted to one another, while similar poles (i.e. two north poles, or two south poles) repel one another. By analogy with electric field lines, magnetic field lines can be imagined to flow from north poles to south poles. At any point where there is a magnetic field, its strength can be defined as the force which would be felt by a small imaginary north pole of unit strength located at that spot. Since there is no such thing as an isolated north pole, what we use to detect a magnetic field is a "compass", i.e. a tiny bar magnet which has both a north and a south pole. In a magnetic field, a compass will not feel any net force (because the force on its north pole will be cancelled out by the force in the opposite direction on its south pole) but in general it will feel a torque as the magnetic forces try to align the compass with the direction of the magnetic field. A magnet, such as a compass needle, is sometimes called a "dipole". Although a single magnetic pole - a north pole, for example - cannot exist in nature, a dipole can. In fact, atoms and molecules often act as magnetic dipoles. It is because of their atomic and molecular structure that ferromagnetic substances can be formed into powerful permanent magnets.

Quantitatively, magnetostatic forces follow the same rules as the forces of electrostatics and of gravitation. The magnitude of the force between two poles is inversely proportional to the square of their distance apart.

A single magnetic pole would produce a magnetic field at every point in its vicinity, the direction of the field being along the line between the pole and the point and its magnitude being inversely proportional to the square of the distance from the pole to the point. The magnetic field due to a set of poles is found just by adding, as vectors, the magnetic field that would be produced by all the poles individually.

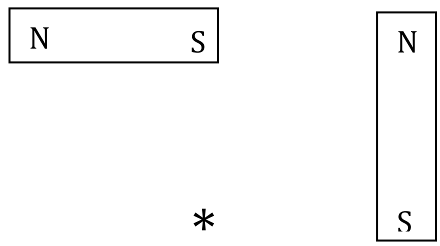
The magnetic field lines (curves, actually) produced by a bar magnet are rather like this:



Question 8l: the magnetic field due to two bar magnets

The diagram below illustrates a pair of identical bar magnets which have been placed on a tabletop. They are pointing at right angles to one another. What is the direction of the magnetic field at the point indicated by the asterisk?

It can be assumed that the distances between the poles of each magnet, and the distance between the south pole of the left-hand magnet and the north pole of the right-hand magnet, and the distances of the asterisk from the poles nearest to it, are all equal.



Question 8m: the force between parallel magnets

We have a pair of identical bar magnets which can be imagined to have poles near to their ends. They are placed on a table parallel to one another and side by side. Show that the force between them would be approximately proportional to the inverse square of their distance apart if that distance were much less than the length of the magnets. If their distance apart were much greater than the length of the magnets, show that the force between them would be approximately proportional to the inverse fourth power of their distance apart.

This question is fairly difficult and you should not worry if you cannot solve it independently.

8.4 Magnetic fields due to moving charges

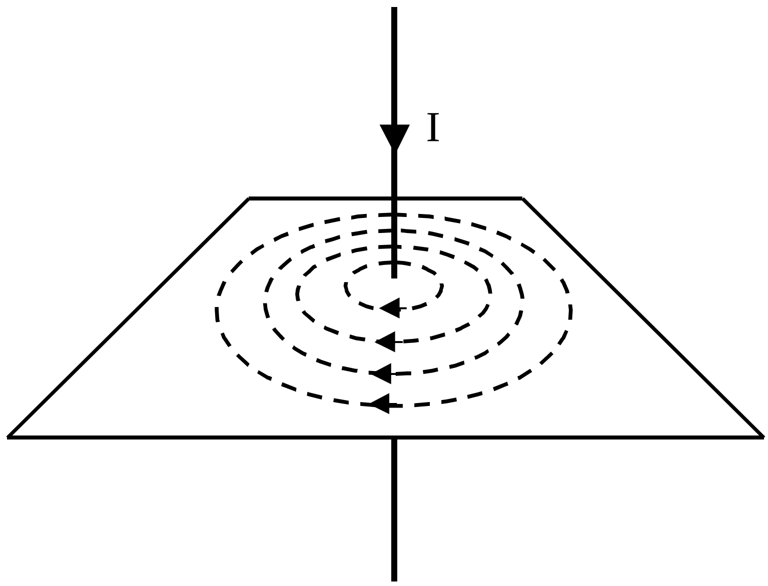
In the previous section, we pretended that magnetic field are produced by magnetic poles in much the same way that electric field are produced by electric charges and gravitational fields are produced by masses.

In fact, the situation is that

Electric charges produce electric fields. Moving electric charges produce magnetic fields as well.

Any kind of magnetic dipole is, in reality, a system within which there are moving electric charges producing what looks like the magnetic field due to a pair of magnetic poles. The Earth itself contains circulating electric currents responsible for the Earth's magnetic field. An atom may form a magnetic dipole if it contains electrons rotating around the central nucleus.

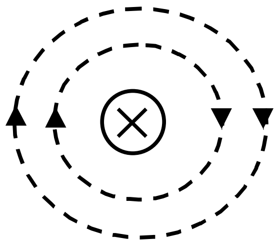
An electric current is a simple system of moving electric charges. Imagine a steady current passing through a long straight wire. The current can be regarded as a stream of moving electric charge. In this situation, it is found that a magnetic field is produced which winds around the wire, with the magnetic field lines consisting of a series of concentric circles spreading outwards from the wire. The direction of the magnetic field will depend upon the direction of current flow. The situation is illustrated here:



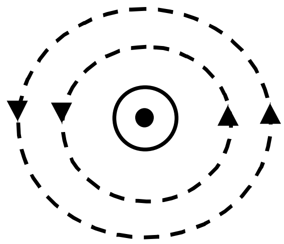
Magnetic field around a straight current-carrying conductor

Maxwell's corkscrew rule

The direction of the magnetic field around a straight current-carrying conductor can be found using the ***corkscrew rule.*** Imagine a corkscrew being driven in the same direction as the current in the conductor. The direction of the magnetic field is the same as the direction of rotation of the corkscrew. The direction of current flow can be denoted by a dot, representing the point of the corkscrew when it is flowing towards the observer, and by a cross, representing the handle of the corkscrew when the current is flowing away from the observer. This is shown below.



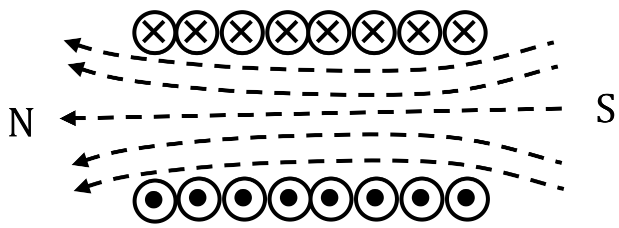
Current flowing away from observer



Current flowing towards observer

Magnetic field of a coil

A important example of the magnetic field produced by a current is the case of a coil (sometimes called a solenoid) in which the wire is wound helically to form a hollow cylinder. The diagram shows a coil sliced by a plane through its axis. The diagram shows just 8 turns of wire, but a real coil might have hundreds of turns.



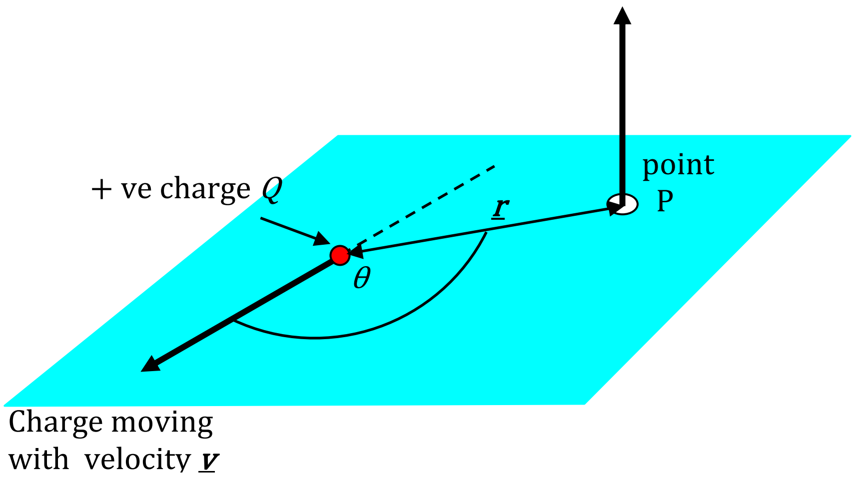
Magnetic field around a current-carrying coil

When a current passes through the wire, and therefore goes around and around the coil, an intense magnetic field can be produced running through the centre of the coil as shown. Seen from outside it, the coil will have the same sort of magnetic field as a strong bar magnet with north and south poles near the letters N and S in the diagram.

The Biot-Savart law

Exactly what is the magnetic field produced by a moving electric charge or current? This question is answered quantitatively by the "Biot-Savart" Law. Suppose that a small charge Q is moving with a velocity $\underline{v}$. What magnetic field does it produce at another point P, if the displacement of P from the moving charge is $\underline{r}$?

The direction of the magnetic field at the point P is perpendicular to the plane containing $\underline{r}$ and $\underline{v}$.



Magnetic field created by a moving charge Q

In the above figure, the blue plane is chosen so that it contains both the vector $\underline{v}$ (the velocity of the moving charge) and also the vector $\underline{r}$ (the separation from the charge Q to the arbitrary point P).

Then, the direction of the magnetic field at P is perpendicular to the blue plane, as illustrated by the vertical arrow in the diagram.

The magnitude of the magnetic field at P is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{Qv\sin(\theta)}{r^2}$$

This equation requires some explanation. First of all, notice that we use the letter B to represent the magnitude of a magnetic field. This is the conventional letter to use for this purpose.

What are the units of this equation? Magnetic field strength, unlike electric field strength, has its own named unit in the SI system. It is the tesla (T).

As one would expect, the magnetic field strength is in proportion to the magnitude Q of the moving charge. It is also in proportion to the speed v. It is also inversely proportional to the square of the distance r, in accord with Coulomb's Law for electric forces and Newton's Law of Gravitation for masses.

The factor sin(θ) means that the electric field would be zero of the velocity $\underline{v}$ were directly towards, or directly away from, the point P. To put it another way: the magnetic field's magnitude is proportional to the component of $\underline{v}$ which is perpendicular to $\underline{r}$.

Finally, what about the factor $\mu_0/4\pi$? This is a constant which sets the overall scale of magnetic forces, and it is analogous to the factor $1/4\pi\varepsilon_0$ occurring in the electrical force law. The quantity μ_0 is known as the "permeability" of free space. It is defined so that it is exactly equal to $4\pi \times 10^{-7}$, and its units are newtons per ampere per ampere ($N\ A^{-2}$).

The equation given above is true when the moving charge is in otherwise empty space, i.e. is in a vacuum. When the surroundings are filled with any other material, the permeability of free space (μ_0) must be replaced by a permeability μ specific to that material.

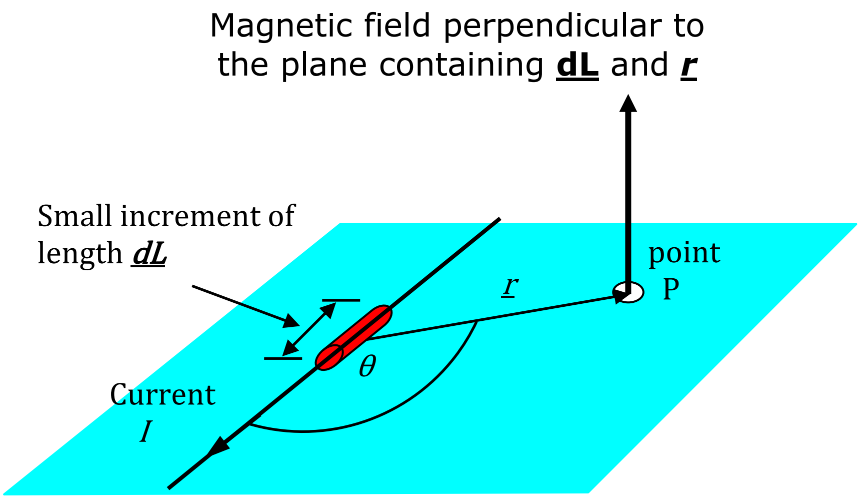
There is a close analogy between permeability μ and permittivity ε. The magnetic permeability of a material is actually a measure of the ease with which the material can be magnetized. Most materials have permeability very close to that of the vacuum, μ_0, but ferromagnetic materials have much higher permeabilities.

Finally, notice that the Biot-Savart law can be written more neatly as a vector equation using the vector "cross" product notation:

$$\underline{B} = \frac{\mu_0}{4\pi} \cdot \frac{Q\ \underline{v} \times \underline{r}}{r^3}$$

The magnetic field produced by a current

Since current flow is moving charge, the Biot-Savart law can easily be written in terms of a steady current I flowing in a small section of a conducting wire. The length of the section of wire is written dL. We treat dL as a vector because its direction, as well as its magnitude, is relevant.



Magnetic field created by a current element *I*dL

Then, *I*dL corresponds to Qv and we get

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dL\sin(\theta)}{r^2}$$

Question 8n

Suppose a current of 1 ampere is flowing around a circular loop of wire which has a radius of 1 metre. What is the magnitude of the magnetic field at the centre of the loop?

Solution: Each part of the wire contributes to the magnetic field according to the formula above. So, for the whole loop,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I\,.2\pi r}{r^2}$$

Now, $\mu_0 = 4\pi \times 10^{-7}$ in the appropriate units, so $\mu_0/4\pi$ is just equal, numerically, to 10^{-7}. The current I is 1 ampere, the radius r is 1 metre, and so the value of B is $2\pi \times 10^{-7}$ tesla, or 0.628 μT.

8.5 Magnetic forces on moving charges and on current-carrying conductors

The Lorentz force equation

At the start of the previous section it was pointed out that electric charges produce electric fields and that moving electric charges produce magnetic fields as well.

Electric and magnetic fields bring about forces. In fact

Electric fields exert forces on electric charges. Magnetic fields exert extra forces on moving electric charges.

So, while electric fields and forces are simply to do with the **presence** of electric charges, magnetic fields and forces are to do with the **movement** of electric charges. If nothing ever moved, magnetism would not exist.

In the previous section we discussed the magnetic field produced by a moving charge: it is given by the Biot-Savart Law. But a moving charge will not only produce a magnetic field: it will also "feel" a magnetic field if one already exists. In other words, a moving charge will experience a force due to any magnetic field through which it is moving. When an electric charge q is stationary, we know that it will experience the force

$$\underline{F} = q\underline{E}$$

where **E** is the prevailing electric field. To take a magnetic field into account, this formula must be extended to

$$\underline{F} = q\,.(\underline{E} + \underline{v} \times \underline{B})$$

where $\underline{v}$ is the velocity of the charge q and $\underline{B}$ is the prevailing electric field. This is known as the Lorentz force equation. Here we have used vector (cross) product notation, and $\underline{v} \times \underline{B}$ is the vector perpendicular to both $\underline{v}$ and $\underline{B}$ and having magnitude vB.sin(θ), where θ is the angle between $\underline{v}$ and $\underline{B}$.

The force on a current-carrying wire in a magnetic field

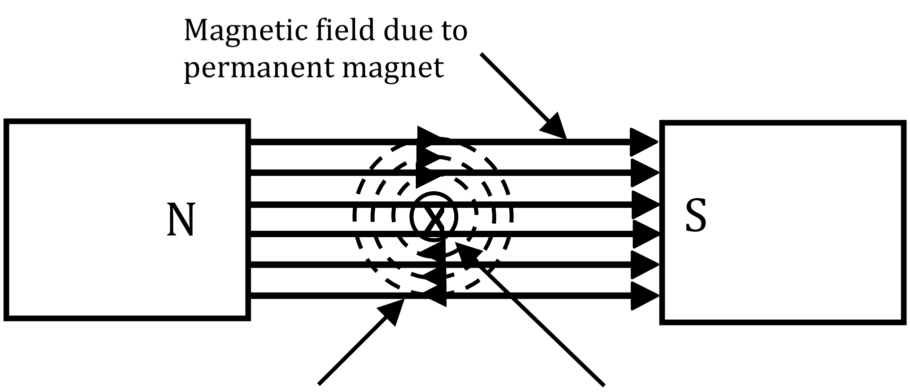
From the Lorentz force equation we can see that an electric charge q moving with velocity $\underline{v}$ in a magnetic field $\underline{B}$, but without any electric field being involved, will experience a force $q\underline{v} \times \underline{B}$.

This means that a current-carrying wire must experience a force in a magnetic field. The movement of a charge q at the velocity $\underline{v}$ is equivalent to a current I moving through a short section of wire with length and direction $\underline{dL}$ where $I\underline{dL} = q\underline{v}$, and the force on the section of wire would therefore be $I\underline{dL} \times \underline{B}$.

> The force F (newtons) on a straight section of wire of length L (metres), perpendicular to a magnetic field of magnitude B (teslas), carrying a current I (amperes) is given by
>
> $$F = BIL$$
>
> but if the wire is not perpendicular but is at an angle θ to the magnetic field, then
>
> $$F = BIL \sin(\theta)$$

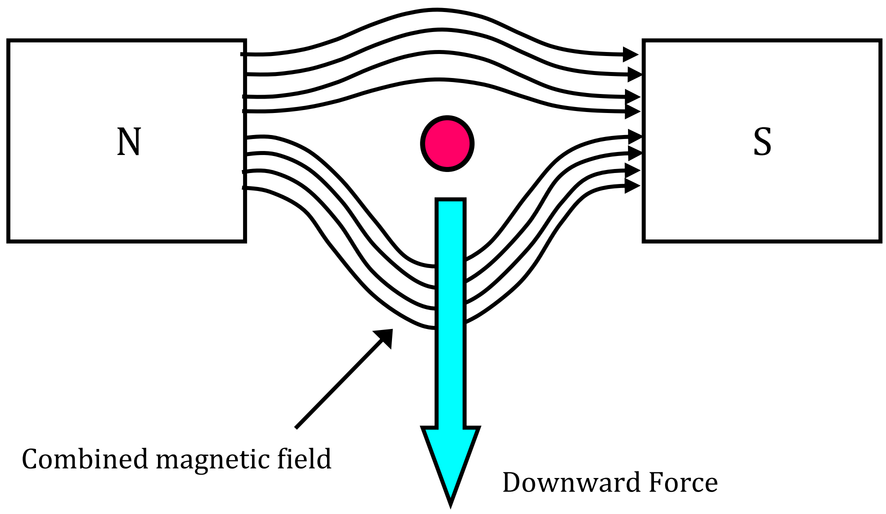
As an alternative way of looking at this phenomenon, consider a straight current carrying conductor placed at right angles to a uniform magnetic field as in the diagram below.

The diagram shows two fields superimposed: the uniform magnetic field lines (left to right) and the magnetic field lines due to the current through the wire (the concentric circles).



Field due to current carrying conductor
at right angles to the magnetic field

The actual magnetic field at each point is the vector sum of those two fields. Above the conductor the lines of magnetic flux are acting in the same direction, and therefore reinforce one another, whereas below the conductor the lines of the magnetic fields are acting in opposite directions and will to some extent cancel each other out. The net magnetic field, therefore, will look something like this:



We can imagine the magnetic field lines to be a bit like threads of elastic which "try" to spread out from one another. So, in this example, the wire will be pushed downwards by the closely-bunched magnetic field lines above it. Of course, this argument is not mathematically rigorous. To work the force out properly, we should just consider the magnetic field which exists without the current (the uniform horizontal field) and then work out $I\underline{dL} \times \underline{B}$. Since the wire and the field are perpendicular, this vector product gives a force which has the magnitude IB per unit length of wire, and is in a vertical direction. To be able to say which way the force goes - up or down - it is convenient to use "Fleming's left-hand rule". This is nicely illustrated in Wikipedia.

To make all this a little clearer, consider the diagrams below, which show how the magnetic force on a wire varies in magnitude as a consequence of the direction of current flow i with respect to the direction of the magnetic field $\underline{B}$.

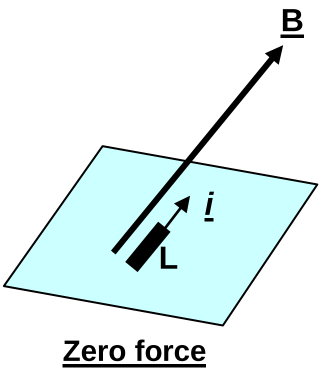


Figure (a): ***PARALLEL***
When the direction of charge movement and B are parallel, the angle between *i* and B is 0^0. Since $sin(0) = 0$, then from equation (1), the magnetic force F = 0 N.

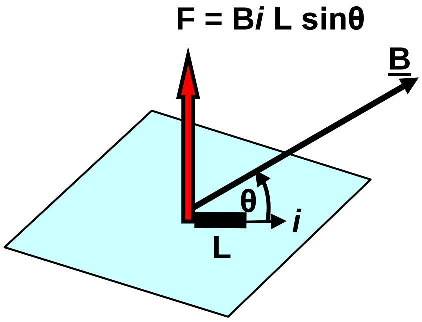


Figure (b):
For angles between 0^0 and 90^0, $0 < sin(\theta) < 1$, therefore from equation (1), the magnetic force $F = BiLsin(\theta)$.

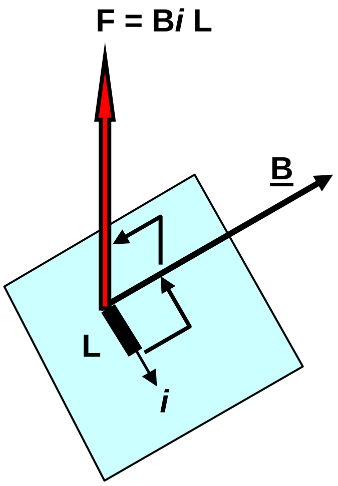
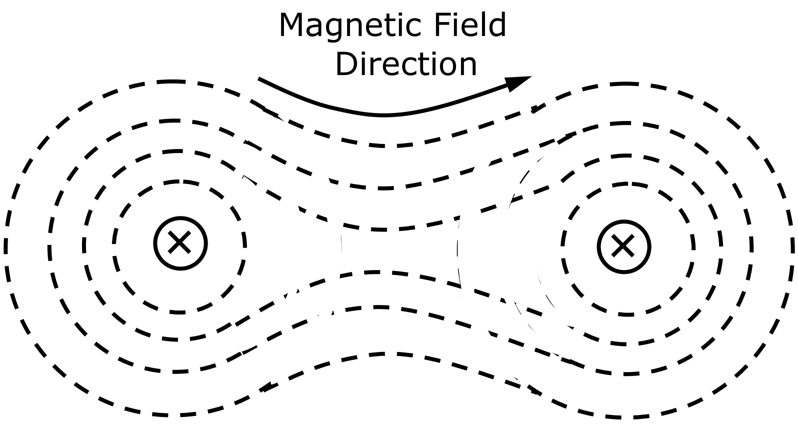


Figure (c): ***PERPENDICULAR***
When the direction of charge movement and B are perpendicular, the angle between *i* and B is 90^0. Since $sin(90°) = 1$, then from equation (1), the magnetic force is at a maximum i.e. $F = BiL$.

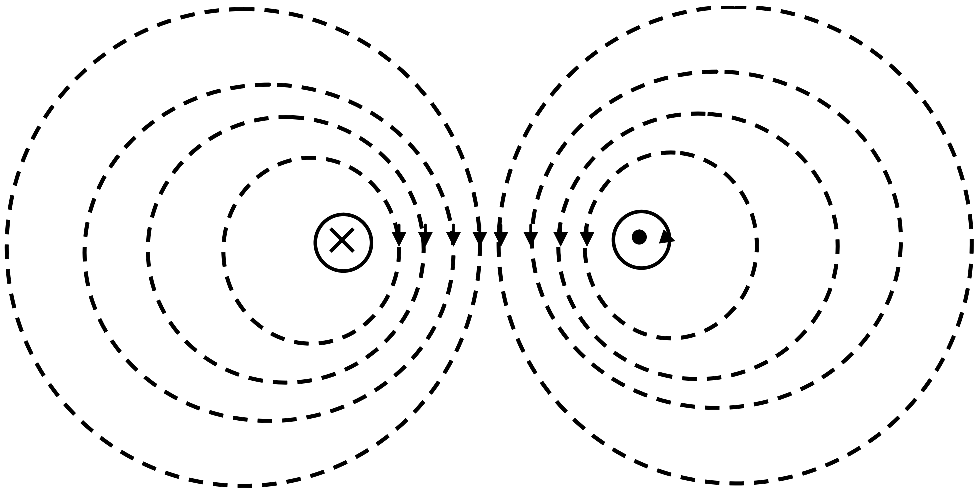
Two parallel current-carrying conductors

An interesting system is a parallel pair of long straight current-carrying wires. If two parallel conductors are carrying a current, each will contribute to the overall magnetic field in the neighbourhood of the wires. The contribution from a single wire will be of the kind considered earlier – a magnetic field with lines in concentric circles around the wire. When there are two wires, the two contributions must be added together as vectors at every point. When the wires are carrying currents in the same direction, the result will look like this:



Currents in the same direction

On the other hand, when the currents are in the opposite directions, the total magnetic field must look like this:



Currents in opposite directions

However, there will also be forces between the wires. Just by looking at the diagrams, and thinking of the magnetic field lines as being like elastic, you might imagine that the wires will be attracted to one another when the currents are in the same direction (parallel) but repelled from one another when the currents are in opposite directions ("antiparallel"). You would be right.

To work out more carefully what is happening, it is necessary to consider the force that would be exerted on the right-hand wire as a result of the magnetic field due to the left-hand wire alone. Then, consider the force that would be exerted n the left-hand wire as a result of the magnetic field due to the right-hand wire alone. In other words, to work out the force on one current-carrying wire you have to consider the magnetic field due to the other wire(s) in the system but excluding the field due to the particular wire in question. These considerations should convince you that, indeed, the wires attract one another when the currents are going in the same direction but repel one another when the currents are in opposite directions.

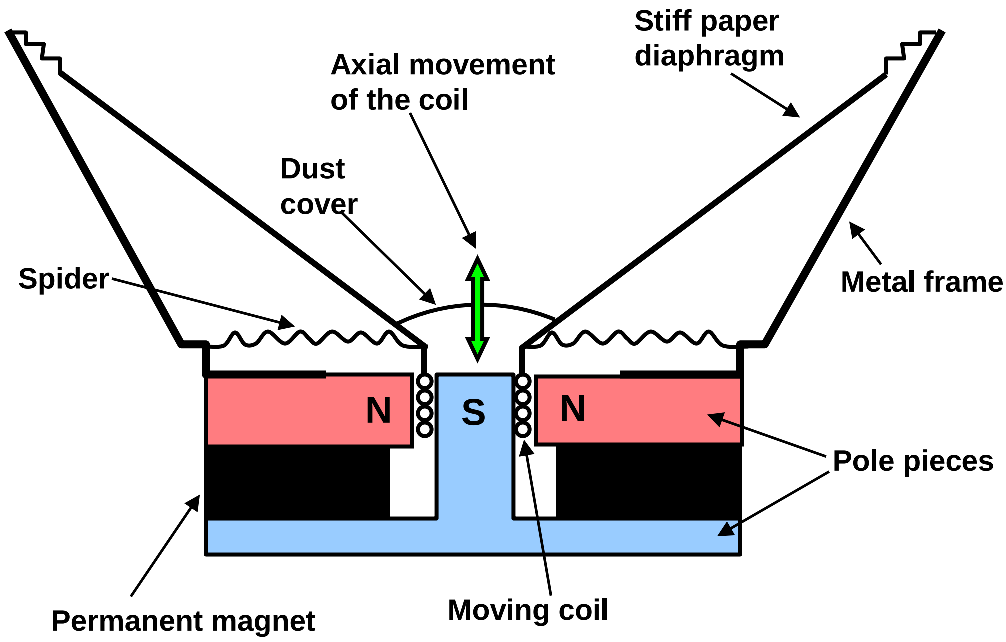
8.6 Magnetic forces on current-carrying conductors: two applications

The Moving-coil loudspeaker

The moving coil loudspeaker consists of a strong permanent magnet and pole pieces to produce a strong radial magnetic field in the air gap. This field is at right angles to a multi-turn coil of copper wire. This coil is able to move axially within the air gap and it is attached to a stiff paper cone or diaphragm. The function of the 'spider' is to prevent the cone moving laterally and to return the coil to its initial position when the coil current is zero.

When an alternating current is passed through the coil, the coil will move in synchronism with the current due to the magnetic force. The movement of the coil is transmitted to the diaphragm which creates the necessary changes in air pressure to produce the sound.

A diagram of a typical moving coil loudspeaker is shown below.

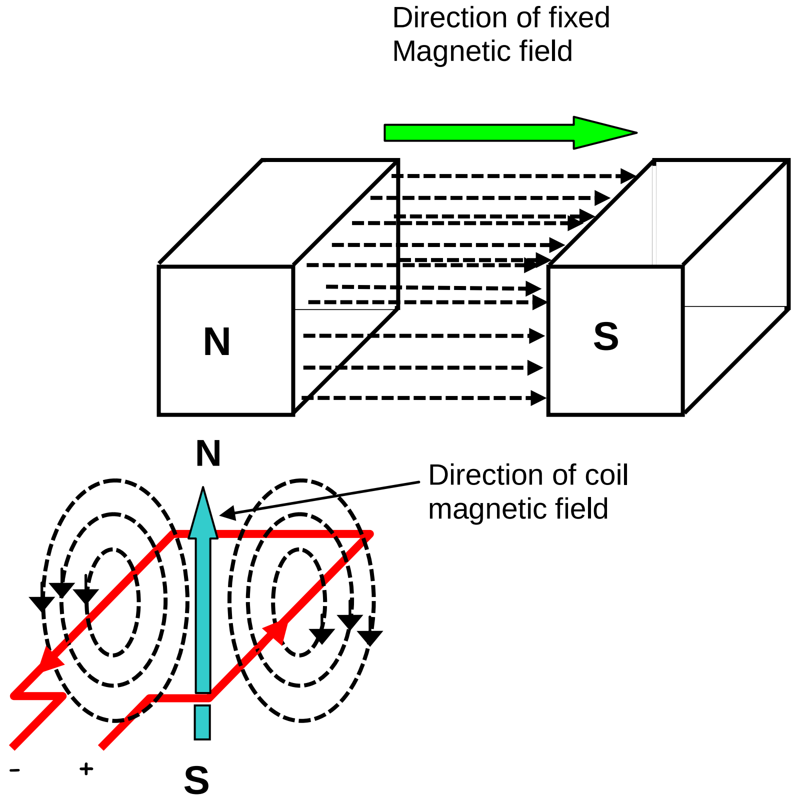


The DC motor

The operation of the DC motor is more complex than that of the moving-coil loudspeaker, since, to produce continuous rotation, the direction of force must be reversed in each of the rotating conductors after every 180° of rotation. The operation of the DC motor can be considered by looking at the interaction between the magnetic field produced by the stationary field system and the rotating coil.

The basic DC motor is based on a stationary magnetic field as shown below. This magnetic field is normally provided by electromagnets. Free to rotate within this magnetic field is the armature winding. A single-turn coil is used to provide a simple armature winding. A DC supply is connected to the ends of this coil. The function of the armature is to have a torque developed on it, causing it to rotate.

With the DC supply to the armature coil as shown, a two pole magnetic field will be produced whose axis will be vertical and having the direction shown (corkscrew rule). When placed between the field poles, the armature coil will rotate clockwise through 90° before the two magnetic fields align.



Continuous rotation can be achieved by reversing the direction of current flow in each armature conductor as it moves from under the influence of the stationary North pole to that of the South pole and vice versa, such that the directions of current shown in the armature above remains the same even though the armature is rotating. The device that performs this switching operation is the commutator. The action of the commutator and its sequence of switching for a simple single turn armature coil are shown in the diagrams below. Note: for convenience the brushes are shown on the inside of the commutator.

In figure 1 the shaded commutator segment is connected to the positive brush and the unshaded commutator segment is connected to the negative of the DC supply. The axis of the armature magnetic field is at right angles to the stationary field, causing the armature to rotate in a clockwise direction.

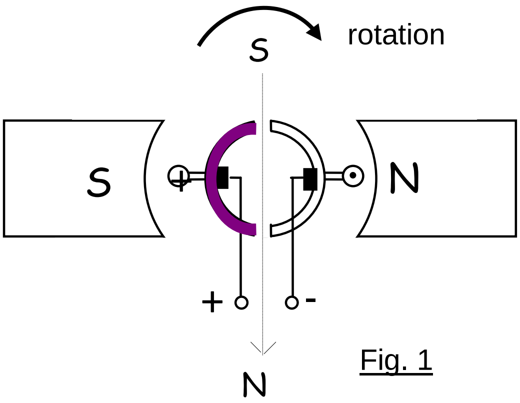


Fig. 1

In figure 2 the armature has turned through 90^{o}. At this point the armature coil is short circuited and the current through the armature coil falls to zero, prior to its direction being reversed. The momentum of the armature carries it beyond this point.

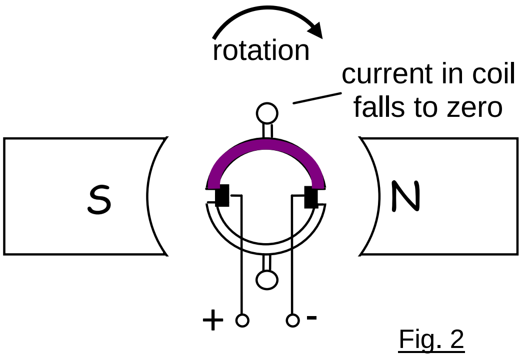


Fig. 2

In figure 3 the unshaded commutator segment is now connected to the positive brush and the shaded commutator is connected to the negative of the DC supply. The axis of the armature is once more at right angles to the stationary magnetic field, causing the armature to continue to rotate in a clockwise direction.

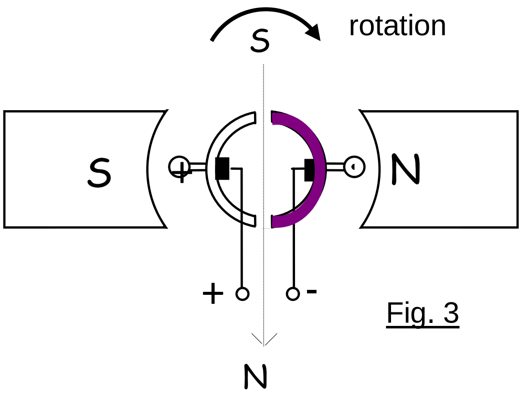


Fig. 3

It can be seen that the commutator, in effect, changes the DC supply at the brushes into an AC current within the armature coil, allowing continuous rotation of the armature.

8.7 Just a little more about electromagnetism

Electromagnetism is a subtle subject which most people find difficult. It calls for a good three-dimensional visual imagination, and a proper mathematical treatment requires vector calculus beyond the scope of this course – the treatment given here is just a simplified introduction.

Electromagnetism involves some tricky concepts. There are electric and magnetic vector fields, which can extend across otherwise empty space. There are the quantities of permittivity and permeability, which are properties of any particular material but are also possessed by a vacuum. By considering the case of a capacitor we have seen that an electric field has an energy density associated with it. In fact, so does a magnetic field, and the energy density at any point *in vacuo* where there are both electric and magnetic fields, $\underline{E}$ and $\underline{B}$, will be

$$½\varepsilon_0\,.(E^2 + \; c^2B^2)$$

ioules per cubic metre. Investigating this in more depth, including the reason why the velocity of light, c, crops up here, is a pleasure we must defer.

It can be difficult to understand that a moving charge (or a current in a wire) not only **creates** a magnetic field, and therefore potentially exerts forces on other parts of the system; but it will also **feel a force** from any pre-existing magnetic field produced by some other part of the system. When thinking about this, it is as well to bear in mind Newton's Third Law of Motion – that, for every force, there is an equal and opposite force – because it is at the heart of how moving charges both create magnetic fields and feel them.

In this section of the course we have introduced a number of new kinds of quantities. Here is a list, including the SI unit names and base units:

Current I	(ampere, A)	[A]
Charge Q	(coulomb, C)	[s A]
Electric field E	(no name)	[N C^{-1} or V m^{-1} or kg m s^{-3} A^{-1}]
Voltage V	(volt, V)	[J C^{-1} or kg m^2 s^{-3} A^{-1}]
Capacitance C	(farad, F)	[C V^{-1} or kg^{-1} m^{-2} s^4 A^2]
Magnetic field B	(tesla, T)	[N m^{-1} A^{-1} or kg s^{-2} A^{-1}]
Permittivity ε	(no name)	[C V^{-1} m^{-1} or kg^{-1} m^{-3} s^4 A^2]
Permeability μ	(no name)	[kg m s^{-2} A^{-2} or T m A^{-1}]

You might think that this is rather daunting, but actually you have been let off lightly: there are many other quantities that we have not attempted to introduce, like resistance, electric displacement, inductance, magnetizing field, magnetic flux, magnetic pole strength,....

We use the term "electromagnetism" because electricity and magnetism are so closely bound up with one another that they should be regarded as aspects of a single type of force. To see this more clearly, imagine a stationary pointlike charge Q. We know that it will produce an electric field and that if it is stationary it will not produce any magnetic field. Now imagine that you are observing the same charge Q but you are sitting in a moving car with velocity $\underline{v}$. From your point of view, the charge Q has a velocity $-\underline{v}$. But a charge moving with a velocity will produce a magnetic field as well as an electric field. So, sitting in your car, you will see the moving charge Q apparently producing a magnetic field.

Well, does the charge Q produce a magnetic field or doesn't it? The answer must be that the charge Q produces an electromagnetic field which, in general, looks different to different observers. And a field which seems purely electric to one observer may seem to have a magnetic component when it is looked at by another observer with a different velocity.

So, when we say that a field is "electric" or "magnetic" we are making a statement which will only be true for particular observers. In general, a field is "electromagnetic" and the extent to which it contains electric and magnetic components will depend on the velocity of the observer!

Question 8o

Near the surface of the Earth, in equatorial regions, the Earth's magnetic field has a strength of approximately 60 μT. What is the energy density contained in this magnetic field?

Solution: use the formula given above,

$$½\varepsilon_0 \,.\, (E^2 + c^2B^2)$$

with $E = 0$ and $B = 60 \times 10^{-6}$ T. The value of ε_0 is 8.854×10^{-12} F m^{-1}, and the velocity of light c is 3×10^8 m s^{-1}. Putting in the numbers, the energy density comes out to be just over 1.4 mJ m^{-3} (millijoules per cubic metre).