Section 1: The SI System of Units

In the past, over a period of many generations, separate sets of units were developed for calculations in different branches of science and technology. In many instances, different countries used different units even in the same branch of science or technology. This used to cause great confusion, and it was therefore eventually decided that a single set of units should be adopted world-wide by all scientists and engineers. This set of units is the "Système Internationale d'Unités", or the "SI System" for short, and it is based on the Napoleonic metric system. The advantages of the system may be summarised as follows :-

- ***Uniformity*** in that units for all quantities which have the same meaning are identical in all branches of science (e.g. the unit of mechanical power is the same as the unit of electrical power, namely the watt).

- ***Coherence*** in that the product or ratio of any two basic units is the unit of the resulting quantity (e.g. energy is power multiplied by time, and therefore the unit of energy (the joule) is equal to the unit of power (the watt) multiplied by the unit of time (the second).

- ***Consistency*** in that there is one basic unit for each physical quantity (e.g. the basic unit of length is the metre - not the inch, or the foot, or the fathom, or anything else).

- ***Manageability*** in that the very large or very small numbers which crop up in different branches of science may be expressed as moderately-sized numbers multiplied by powers of 10.

Basic Units: There are seven basic SI units from which all other units are derived :-

QUANTITY	UNIT NAME	SYMBOL
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	kelvin	K
Luminous Intensity	candela	cd
Amount of Substance	mole	mol

Note :- The unit of temperature favoured for everyday use is the degree Celsius, which is the same size as the degree Kelvin. One degree Celsius (C) is approximately equal to one degree Centigrade (°C).

ALL OTHER SI UNITS MAY BE DERIVED FROM THESE SEVEN.

Supplementary Units:

QUANTITY	UNIT NAME	SYMBOL
Plane angle	Radian	rad
Solid Angle	Steradian	sr

1.1 Units of Length

The basic unit of length is the metre (m), and common multiples of this are used. These multiples are in the form of :-

kilo meaning one thousand times the basic unit
deci meaning one tenth (1/10) of the basic unit
centi meaning one hundredth (1/100) of the basic unit
milli meaning one thousandth (1/1000) of the basic unit

From the above, therefore:

1 kilometre (km) = 1000 metres
1 decimetre (dm) = 1/10 metre (m) = 0.1 m
(or 10 dm = 1 m)
1 centimetre (cm) = 1/100 m = 0.01 m
(or 100 cm = 1 m)
1 millimetre (mm) = 1/1000 m = 0.001 m
(or 1000 mm = 1 m)

To convert from	*to*	*do the following*
dm	m	divide by 10
cm	m	divide by 100
mm	m	divide by 1000
m	dm	multiply by 10
m	cm	multiply by 100
m	mm	multiply by 1000

Example: 3.2 m expressed as mm is 3.2 x 1000 = 3200 mm

A quick method is to insert the decimal point, and move it 3 places to the right:

3 . 2 0 0. = 3200 mm
→→→

***Remember, when converting from large multiples of units to small multiples of units, the number must get* BIGGER.**

Example: 2521 mm expressed as m is 2521/1000 = 2.521 m

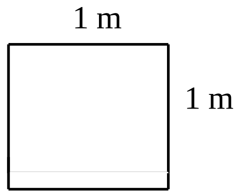
A quick method here is to insert the decimal point and move it 3 places to the left:

2 . 5 2 1 . 0 = 2.521 m
←←←

***Remember, when converting from small multiples of units to large multiples of units, the number must get* SMALLER.**

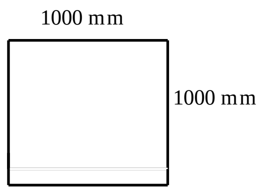
1.2 Units of Area

The unit of area is the m^2 (not "sq m").



Area = 1 m x 1 m = 1 m^2

Likewise:



Area = 1000 mm x 1000 mm
= 1000000 mm^2
= 1 million mm^2

This may be written as 1×10^6 mm^2

So, 1 m^2 = 1000000 mm^2 = 1×10^6 mm^2

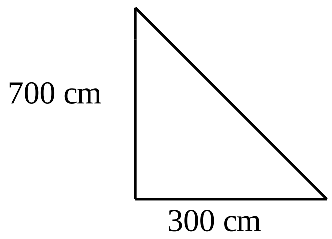
In a similar way, 1 m^2 = 10 dm x 10 dm = 100 dm^2
(or 1 dm^2 = 1/100 m^2)

Also 1 m^2 = 100 cm x 100 cm = 10000 cm^2

(or 1 cm^2 = 1/10000 m^2)

Question 1a

Determine the area (in m^2) of the triangle shown, given that the area of a triangle is given by the formula (base $\times$ height)/2

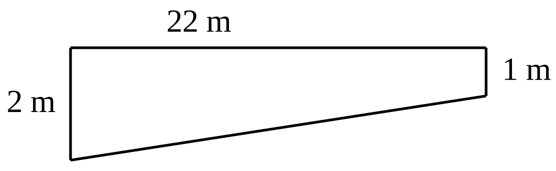


Solution: first convert the distances into metres, getting 0.7 m for the height and 0.3 m for the base. Half their product is 0.105, which is the required area in units of m^2.

Question 1b

The figure shown below is a trapezium. Determine the area from the equation:

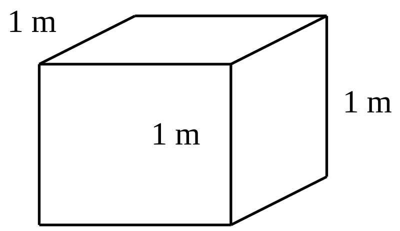
area = (average of sides) x length



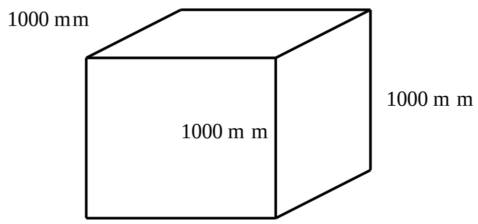
Solution: One side has a length of 1 m, and the other 2 m. Their average is 1.5 m. This average, multiplied by the length of 22 m, gives the area as 33 m^2.

1.3 Units of Volume

The basic unit of volume is the m^3 (not the "cubic metre")



Volume = 1 m x 1 m x 1 m = 1 m^3.



Volume = 1000 mm x 1000 mm x 1000 mm
= 1 000 000 000 mm^3 (or 1 x 10^9 mm^3)

Similarly, 1 m^3 = 10 x 10 x 10 = 1000 dm^3
(or 1 dm^3 = 1/1000 m^3)
1m^3 = 100 x 100 x 100 = 1 000 000 cm^3
(or 1 dm^3 = 1/1000000 m^3)

Note : although 1 m^3 is the standard unit of volume, it has been accepted that for everyday use, the dm^3 is more convenient. This is called a **litre.**

So, 1 dm^3 = 1 litre, and as 1000 dm^3 = 1m^3, 1000 litres = 1 m^3. Also, as 1000 000 cm^3 = 1 m^3, so 1000 cm^3 = 1 litre. Notice that 1 cm^3 may be called 1 millilitre (ml).

Question 1c

What is the capacity in litres of a rectangular tank having the dimensions 4m x 3m x 2m?

Solution: the volume of such a tank is the product of its length, breadth and height. So, the volume is 4 x 3 x 2 = 24 m^3.

1.4 Units of Mass

The basic unit of mass is the kilogram (kg), and a common multiple of this is the megagram (Mg). A much more common name for the megagram is the metric tonne (note the spelling). The tonne is a little bit smaller than the Imperial ton.

A convenient sub-multiple of the kilogram is the gram (g):

1 kilogram = 1000 g so 1 g = 1/1000 kg

1 Mg or 1 tonne = 1000 kg

1.5 Units of Density

Density is defined as the mass per unit volume, in other words it is the mass of a quantity of the substance which occupies a volume of $1m^3$.

Density = mass divided by volume:

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

So, if an object has a mass of 1 kg and a volume of $1m^3$, then its density is 1 kg m^{-3}.

The unit of density in the SI system does not have a special name of its own: it is simply written as kg m^{-3} and may be pronounced as "kilograms per cubic metre".

Density of Water: a litre of water has a mass very close to one kilogram. So, the density of water is 1 kg litre^{-1}. As 1000 litres = 1 m^3, the density of water may be expressed as 1000 1000 kg m^{-3}.

Question 1d

A rectangular block of steel measures 200 mm x 100 mm x 60 mm and it has a mass of 9.42 kg.   Calculate its density.

Solution: the block's volume is 0.2 x 0.1 x 0.06 = 0.0012 m^3. Dividing this into the mass, the density comews out as 7850 kg m^{-3}.

Question 1e

A rectangular tank is 22 m long and 10 m wide. What depth of water is contained in it if the mass of the water is found to be 660 tonnes?

Solution: the mass of the water is given by the equation

$$mass = volume \times density$$

$$= depth \times length \times width \times density$$

So,

$$depth = mass / (length \times width \times density)$$

$$= (660 \times 10^3\ kg) / (22\ m \times 10\ m \times 1000\ kg\ m^{-3})$$

$$= 3\ m$$

1.6 Units of Speed and Velocity

In standard SI units, speed and velocity are measured in metres per second, m s^{-1}, but the non-standard unit km h^{-1}, "kilometres per hour", is often encountered.

The speed of a body can be defined loosely as the rate of change of its position - in other words, how far it moves in one second or in one hour. Speed has no particular direction, and is always a positive quantity.

If movement is taking place in a specified direction, or relative to a specified co-ordinate system, the term "velocity" is more appropriate. For example, in one dimension, if the position of a body is measured using a coordinate x, then a "velocity" of 1 m s^{-1} means that the value of x is increasing by 1 m every second. In such a situation, velocity may be either positive (x increasing, movement forewards) or negative (x decreasing, movement backwards).

Question 1f

If a train travels a distance of 600 m, at a steady speed, in 5 seconds, what is its speed?

Solution: speed = distance/time = 600/5 = 120 m s^{-1}

Question 1g

Express the above speed in kilometres per hour.

Solution: converting between units is always tricky and should be done with care. There are 3600 seconds in an hour, so a train which travels 120 metres per second will travel 120 x 3600 metres per hour. Every 1000 metres makes a kilometre, so the train travels 120 x 3600/1000 kilometres per hour. So, the speed can be expressed as 432 km h^{-1}.

Question 1h

If a car travels at a constant speed of 20 km h^{-1}, for 5 minutes, find the distance travelled.

Solution: in general, ***distance = speed x time***.

The safest way to answer this question is first to express both the speed and the time in standard SI units:

$$speed = 20 \ km\ h^{-1} = 20000 \ m\ h^{-1} = 20000/3600 \ m\ s^{-1}$$

$$time = 5 \text{ minutes} = 300 \text{ seconds}$$

Then,

$$distance = (20000/3600) \times 300 = 20000/12 = 1667 \ m$$

Question 1i

If a train travels a distance of 100 km in 1 hour 40 minutes, what is its average speed in m s^{-1}?

Solution: travelling 100000 metres in 6000 seconds corresponds to the speed 100000/6000 = 16.7 m s^{-1}.

1.7 Units of Acceleration

Acceleration is defined as the rate of change of velocity. In other words, the acceleration of a body is a measure of how much the body's velocity changes in each second.

The units of acceleration are m s^{-1} per second, making m s^{-2}.

Question 1j

A car starts from rest and accelerates uniformly in a straight line for 3 seconds at 2 m s^{-2}. Find its speed after 3 seconds**.**

Solution: after 1 second its speed will be 2 m s^{-1}. After 2 seconds its speed will be 2 + 2 = 4 m s^{-1}. After 3 seconds it will be 4 + 2 = 6 m s^{-1}.

In general, if a vehicle travels in a straight line with uniform acceleration, then its final velocity will be equal to its initial velocity plus the acceleration multiplied by the time of the acceleration:

final velocity = initial velocity + acceleration x time

So this question could have been answered by calculating

final velocity = 0 + 2 x 3 = 6 m s^{-1}.

If an object is slowing down (or speeding up but in a backwards direction!) then it is said to have negative acceleration (or "retardation") and the acceleration could be written as a negative number.

Question 1k

If a car decelerates uniformly at 5 m s^{-2} from a speed of 20 m s^{-1}, find its speed after 3 seconds.

Solution: final velocity = initial velocity + acceleration x time

$$= 20 + (-5) \times 3 = 20 - 15 = 5 \text{ m s}^{-1}$$

Question 1l

If a train increases its speed from 10 m s^{-1} with a uniform acceleration of 5 m s^{-2} for 5 seconds**,** calculate its final speed.

Solution: 10 + 5 x 5 = 10 + 25 = 35 m s^{-1}

Question 1m

If the train above now slows down at a uniform rate of 5 m s^{-2}, how long will it take to stop?

Solution: *final velocity = initial velocity + acceleration x time, so if the final velocity is zero then we must have*

initial velocity = (- acceleration) x time

or

time = (initial velocity)/(- acceleration)

In this example, the train is slowing down so its acceleration is negative. Therefore, "(- acceleration)" is positive. The initial velocity is what was calculated to be the "final speed" in the previous question. So, the time is simply 35/5 = 7 s.

The Acceleration Due to Gravity (g)

If you get tired of working your way through this material, and decide to drop your pen out of the window, it will accelerate towards the ground at the rate of 9.81 m s^{-2}. This is the acceleration *g* of any object in free fall due to the Earth's gravitational force. The acceleration due to gravity is *independent of the mass*, so that if two pens of different mass were dropped out of the window at the same time they would hit the ground together (as long as we can ignore the effect of air resistance). The acceleration due to gravity on the surface of the Moon is about one-sixth of that on Earth. Under some circumstances, to make calculations easier, g is approximated as 10 m s^{-2}.

1.8 Units of Force

Acceleration results from the application of a force to a body. Isaac Newton's Second Law of Motion can be expressed as:

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

Obviously the units of force are the units of mass multiplied by the units of acceleration. So, the units of force are kg m s^{-2}. However, in the SI system the unit of force has a special name, the newton, and it is abbreviated to N. One newton, therefore, is the force required to give a mass of 1 kg an acceleration of 1 m s^{-2}.

Multiples of newton's may be written as follows:

1 000 N = 1 kilonewton (kN) = 1×10^3 N
1 000 kN = 1 meganewton (MN) = 1×10^6 N
1 000 MN = 1 giganewton (GN) = 1×10^9 N

In the case of the Earth's gravitational attraction, the force acting on a body of mass m is mg. This force is the "weight" of the body.

"Weight" is the force with which a body is attracted towards the Earth.

The Distinction between Mass and Weight

Mass is a measure of the quantity of matter that a body contains, and it is not affected by gravity.

Weight is a force dependent on the acceleration due to gravity, and in general it varies from place to place.

1.9 Units of Pressure

Pressure is force per unit area, with units of newtons per square metre. The unit of pressure is given a special name, the pascal (abbreviation Pa), and

1 pascal (Pa) = 1 newton per square metre (N m^{-2})

Note that a pascal is a rather small unit (roughly the weight exerted by 0.1 kg over an area of one square metre). Atmospheric pressure is about 100 000 Pa = 10^5 Pa. Common multiples of the pascal are the megapascal (1 MPa = 10^6 Pa) and the gigapascal (1 GPa = 10^9 Pa).

1.10 Units of Momentum

The momentum of a body is the product of its mass *m* and its velocity *v*. It follows that momentum *mv* has units of kg m s^{-1}.

Since as newton has the fundamental units kg m s^{-2}, the units of momentum can be expressed as newton-seconds, N s.

The unit of momentum has no special name.

1.11 Energy

We do work on an object when we exert a force on it over a certain distance. "Work" can be defined as the force and the distance over which the force acts. For example, if I lift an object of mass *m* through a height *h*, then I would have to do an amount of work

work = force x distance

= weight x height

= *mgh*.

The units of work can therefore be represented as N m or as kg m^2 s^{-2}. This unit is given a special name, the joule, abbvreviated to J. So:

1 joule (J) = 1 newton-metre (N m) = 1 kg m^2 s^{-2}

Energy is the "potential for doing work": by raising the mass *m* through the height *h* we have increased its gravitational potential energy by an amount *mgh*. Energy, therefore, has the same units as work.

1.12 Power

Power is defined as the rate at which work is done or energy is supplied. The unit of power is the joule per second ($J\ s^{-1}$). This has the special name of the watt (W). So:

$$\mathbf{1\ \ watt\ (W)\ =\ 1\ \ joule\ per\ second\ (J\ s^{-1})}$$

$$\mathbf{=\ 1\ \ newton\text{-}metre/second\ (N\ m\ s^{-1})\ =\ 1\ \ kg\ m^2\ s^{-3}}$$

Commonly encountered multiples are the kilowatt (1 kW = 10^3 watts), the megawatt (1 MW = 10^6 watts) and the gigawatt (1 GW = 10^9 watts). A very large power station would be rated at about 1 GW.

Question 1n

If a 1 GW power station runs non-stop for a year, what is the total energy it produces in that time?

Solution: energy = power x time. In this case, the time is one year, which is approximately equal to 365 x 24 x 60 x 60 seconds. The power, 1 GW, is 10^9 watts. So, the energy is 365 x 24 x 60 x 60 x 10^9 joules, which works out to be 31.5 x 10^{15} J.

Summary of prefixes for powers of ten

Standard prefixes are used for units multiplied or divided by factors of 1000. We have met some of these already. A list is given in the table below.

Multiplying factor	Prefix
10^{18}	exa, E
10^{15}	peta, P
10^{12}	tera, T
10^{9}	giga, G
10^{6}	mega, M
10^{3}	kilo, k
10^{-3}	milli, m
10^{-6}	micro, μ
10^{-9}	nano, n
10^{-12}	pico, p
10^{-15}	femto, f
10^{-18}	atto, a

So, the answer to the previous question could be expressed as 31.5 PJ

Conventions about the naming of units

Where a unit is named after an individual (such as Newton, Joule, Watt, Ampère, Volt, Coulomb...), the unit is written with a lower case initial (e.g. newton, joule, watt, ampere, volt, coulomb…) in order to distinguish between the unit and the person. However, when abbreviated, the unit is in upper case (N, J, W, A, V, C...).