Session 14

Heat

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Contents

Welcome

Welcome to session 14 of this course. In this session we'll begin our study of thermodynamics by looking at some of the properties of heat.

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Learning Objectives

- Define temperature and thermal equilibrium
- Define and use specific heats
- Define and calculate thermal expansion
- Explain what is meant by an equation of state
- Describe changes of phase and the associated latent heats
- Compare conduction, convection and radiation in the transport of heat
- Quote and use the Stephan-Boltzmann T^4 law for the rate of radiative heat loss from a blackbody

The Problem

Article 1: Water company bosses plan to tow icebergs up Thames

Transporting water by road or via a national grid has been rejected because of the scale of the shortages and the costs, by Lewis Smith:

"TOWING icebergs from the Arctic is among the measures proposed by water chiefs to solve emergency shortages. The prospect of icebergs sailing up the Thames Estuary to keep London and the Home Counties supplied with tap water seems far-fetched but is under serious consideration by Thames Water.

It is one of the options being looked at as the South East faces the possibility of its worst drought in a century. "

From: http://www.timesonline.co.uk/tol/news/uk/article719902.ece?token=null&offset=0

Image[1]

[1] Ilulissat Iceberg and Esle, Grenland, by kaet44, as posted on www.flickr.com. Creative Commons Licensed.

Article 2:

"Australian polar scientist Professor Patrick Quilty thinks he has a pretty cool idea.

He wants to move Antarctic icebergs around the world for use as a source of water.

Yes, icebergs.

Professor Quilty reckons it can be done by wrapping icebergs in huge, and he means HUGE, plastic bags and towing them to places like Africa where water is a scarce commodity. "

From: http://www.abc.net.au/news/features/antarctica/

Image[2]

These two news articles suggest that icebergs can be towed to drought stricken coastal or river areas to provide fresh water. Icebergs are regularly towed to prevent collisions with drilling platforms so the technical problems are not insurmountable. The question that arises from an economic point of view is how much of the iceberg would survive melting. A slightly simpler but equivalent question is how long would it take the iceberg to melt completely? So that's what we'll look at.

[2] Floating Iceberg, by Alan Vernon, as posted on www.flickr.com. Creative Commons Licensed

How long does it take to melt an iceberg?

What are the learning issues for this problem? Think about yours before going on.

What will affect the time it takes for an iceberg to melt?

Here are some of ours. Why does ice float? In particular, will it continue to float in fresh water? How much of the iceberg is exposed to the Sun? How much heat does it take to warm up the ice, melt it and evaporate it? Will this heat come from the sea or from solar radiation? In other words, how important are convection and conduction compared to radiation? How much radiation will the ice absorb? How much will it re-radiate? Finally, at a fundamental level, what do we mean by temperature and thermal equilibrium. How can we treat heat flows in bodies that are not in thermal equilibrium?

Facts about icebergs:

Let's start with some facts about icebergs. So as not to complicate the issue we shall focus the problem on one particular iceberg, one with a fairly small one but probably towable mass. Our iceberg has an exposed height above water of 10m and an area of 1000 m^2. A typical speed of such an iceberg driven by ocean currents is 0.2 m s^{-1}, with a mass of 10,000 tonnes.

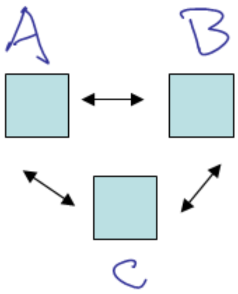
Image[3]

[3] Humpback Whale Tail and Iceberg in Labrador, by natalielucier, as posted on www.flickr.com. Creative Commons Licensed.

Thermal Equilibrium and temperature

A system is in equilibrium if there are no net flows. For example, in mechanical equilibrium there are no accelerations. In thermal equilibrium there is no net heat flow.

Zeroth Law: If a body A is in thermal equilibrium with a body B and body B is in thermal equilibrium with body C then body A is in thermal equilibrium with C



This statement is often called the zeroth law of thermodynamics. It means that we can chose a property of some reference body to determine which systems are in equilibrium – call the property that is recorded by that reference body ‘temperature’ (on that scale). Examples are the volume of a mass of mercury, the volume of a gas at constant pressure, the density of a liquid

The Galilean thermometer works on the basis of temperature dependence of density of a fluid, so the different weights float at different levels. This section shows a picture of one. Find out how they work if you’re not already familiar with the idea.

Image[4]

Temperature and Equations of State

For convenience we choose some reference points and interpolate linearly in between. In the centigrade scale the fixed points are, of course, the boiling and freezing point of water. However, between these points different types of thermometer will agree only if the chosen properties change linearly with temperature.

The perfect gas scale is obtained by measuring changes in pressure P of a gas at fixed volume V as the temperature T is varied. Well above the point at which it liquefies, to a good approximation the gas obeys the equation of state:

$$PV = nRT$$

[4] Death in a temperate zone, by mugley, as posted on www.flickr.com. Creative Commons Licensed.

where $R = 8.31\ J{\cdot}K^{-1}{\cdot}mol^{-1}$ is the gas constant per mole and n is the number of moles.

An equation of state is a relation between the macroscopic variables that define the thermodynamic state of the system.

The perfect gas temperature is also the absolute or thermodynamic temperature, which is defined in terms of the internal energy of the gas particles such that a mole of gas at absolute temperature T has an energy 3/2RT.

This gives the absolute (or thermodynamic) temperature in which absolute zero is about -273 C.

What is a mole? Convert *R* to $N{\cdot}m{\cdot}kg^{-1}{\cdot}K^{-1}$

(Answer: 286.9)

Show that $R = N_0 k$ where N_0 is Avogadro's number, the number of atoms in a mole

Specific heats

How does the change in temperature of a substance change in relation to the heat supplied? This is given by the specific heat, which is defined as the heat absorbed by a unit mass of a substance per unit change in temperature. Equivalently it is the rate of change of heat, Q, with respect to temperature T. Clearly, substances with high specific heats, like water, absorb a relatively large amount of heat for a small change in temperature, whereas substances with relatively low specific heats, like air, store relatively little heat.

$$C = \frac{\Delta Q}{\Delta T} \quad \text{or} \quad \frac{dQ}{dT} \qquad \textit{(Specific Heat)}$$

Example: The specific heat of water is 4.2 kJ $kg^{-1}K^{-1}$. To raise temperature of the 2L of water in a kettle from room temperature (15 C) to boiling takes 4.2 x 2 x 85 kJ = 714 kJ. A 100% efficient 2kW kettle should achieve this in 6 minutes.

Note: The specific heat of a mole of perfect gas at constant volume = 3/2 R.

The constancy of the specific heat of an ideal gas is one way in which the thermodynamic temperature scale is defined. It can be shown to be the same as the perfect gas temperature scale. In other words, a perfect gas thermometer is a realisation of the absolute temperature (at least in as much as real gases approximate perfect gases).

Thermal expansion

Ice floats because it is less dense than water. We therefore look next at the way bodies expand or contract on heating, thereby changing their densities.

The coefficient of expansion of a volume V is defined as:

$$\frac{1}{V}\frac{dV}{dT}$$ *(Volume Expansion)*

It therefore has units of inverse temperature. It's the fractional change in volume per degree change in temperature. Defining it in this way – as the logarithmic derivative, dlogV/dT -- means that it is independent of the quantity of material we start with, since doubling the initial volume leaves the coefficient unchanged.

For linear expansion, like a nail or a railway line, we define the coefficient of thermal expansion as the change in length per unit length per degree.

$$\frac{1}{L}\frac{dL}{dT}$$ *(Linear Expansion)*

So $\Delta V = \alpha V \Delta T$ and $\Delta L = \alpha L \Delta T$

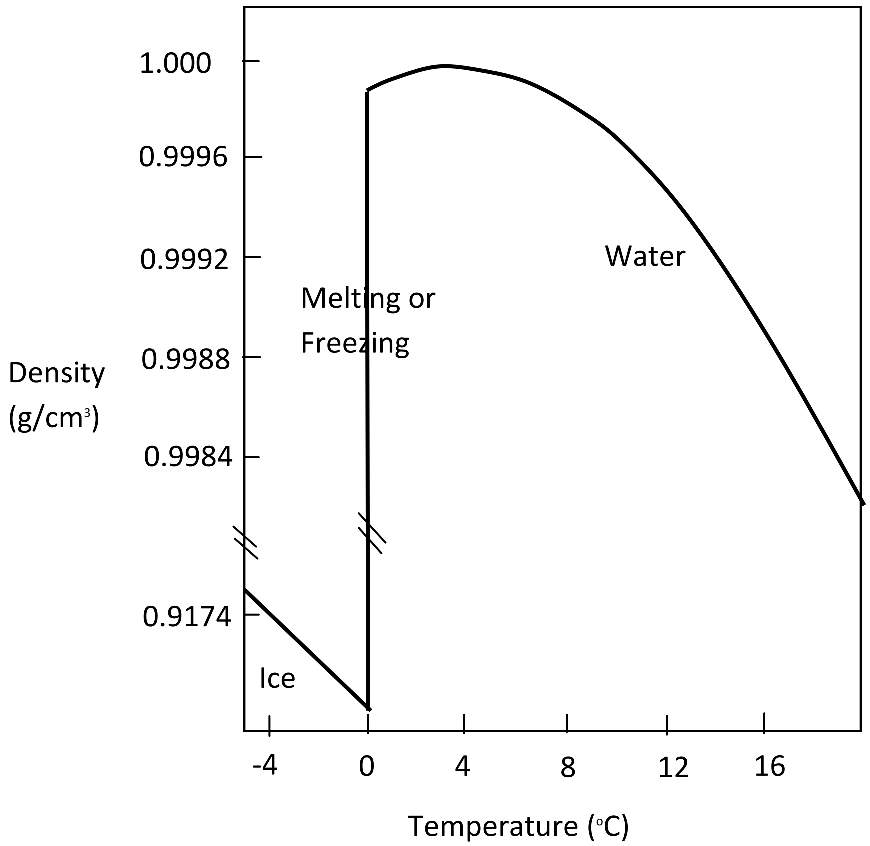
Let's start with the more common behaviour. Our first problem involves a substance, mercury, which expands on heating. Have a go at the problem before continuing with our solution.

Volumetric thermal expansion coefficient for mercury at room temperature is about 1.8 x 10^{-4} K^{-1}. What is the bore of a typical mercury thermometer containing 0.1 cm^3 of mercury?

Answer: Let's take a 10^0 change to correspond to 1cm rise in the column. You'll obviously get a different answer if you made a different assumption. Then the change in volume can be calculated in two ways that must be equal. The change in volume of column = change in volume from expansion. The change in volume is the change in height times the area of the column. If we work in metres a 1 cm rise corresponds to a volume change of 0.01 times the area of the column A. This equals the expansion in volume which is the coefficient of expansion, alpha, times the volume, 10^{-7} m^3 times the change in temperature of 10^o C or 1.8 x 10^{-4} x10^{-7} x 10, giving a bore of 1.8 x 10^{-8} m^2 or somewhat under 1/10th of a mm in radius.

Why does ice float?

Ice is less dense than water so it floats. However, we haven't answered the question as to why ice is less dense than water. The graph shows that, unlike most substances, over a limited temperature range around its freezing point, water expands on cooling from a maximum density at 4^oC.



See also http://www.lsbu.ac.uk/water/explan2.html

To work out how much of the iceberg is submerged we need Archimedes principle: the upthrust on a body floating in a fluid equals the mass of the displaced fluid. From this we can deduce that 90% of the iceberg is below water:

Density of ice = 917 kg m^{-3}

Density of water = 1000 kg m^{-3}.

Archimedes: $V_w\rho_w = V_i\rho_i$

So the ratio of volumes is 917/1000 = 0.9 so 90% of the iceberg is under water.

Summary

- Bodies in thermal equilibrium can be assigned a temperature
- The specific heat of a body is defined as the amount of heat absorbed per unit rise in temperature
- The coefficient of thermal expansion is the fractional change in dimension per unit change in temperature
- The perfect gas equation of state is PV = nRT
- Archimedes Principle states that the upthrust on a floating body equals the weight of fluid displaced.

SAQs

1. A car tyre is inflated to 2 atmospheres (so has a total pressure of 3 atm.) on a hot day. Estimate the type pressure *reading* after a cold night. (Tick the value nearest your estimate)
   (a) 1.7 at
   (b) 2.7 at
   (c) 2.3 at
   (d) none of these

2. Why is the specific heat of paraffin significantly less than that of water?
   (a) paraffin is less dense
   (b) paraffin is inflammable
   (c) paraffin is a non-polar molecule so the is less binding energy between molecules than for water

3. Expansion of railway lines in summer:
   The railway line between Alice Springs and Darwin 1420km apart is continuously welded high tensile steel with a coefficient of expansion of $1.5x10^{-5}$ per degree. The temperature range at Darwin is $45^{o}C$ and at Alice Springs $74^{o}C$. Taking an average along the track, by how much would the length of track change between the hottest day and coldest night? [The rails do not expand because they are pre-stressed and held in place.]
   (a) 309 m
   (b) 21 m
   (c) 1.6 km
   (d) 1.3 km

4. Suppose you were to be wrapped in an insulating blanket. Assuming that you weigh 70 kg and lose energy at a rate of 80W (the so-called basal metabolic rate or BMR) how much approximately would your body temperature rise in one hour. The specific heat of the human body is 3.49 kJ $kg^{-1}K^{-1}$.
   (a) 1K
   (b) 10 K
   (c) 100 K
   (d) 10000 K

The answers appear on the following page

Answers

1. (a) Correct: We have taken the day time temperature to be 30 C and the cold morning temperature to be 0 C. Using the equation of state for a perfect gas the ratio of total pressures is then given by P1/P2 = T2/T1 = 303/273. If the tyre is inflated to 2 atmospheres above atmospheric pressure during the day P2 = 3x273/303 = 2.7 so the pressure gauge will read 1.7 above atmospheric. Any reasonable assumptions will give approximately the same answer.
   (b) Incorrect. You have forgotten that the reading on a pressure gauge is the pressure above atmospheric (at atmospheric pressure the gauge reads 0 and the tyre is not inflated at all.
   (c) Incorrect. You've got the ratio the wrong way up: the pressure must be less at night when it is cooler.
   (d) Incorrect, unless your car has very non-standard tyres. A common mistake is to use temperature in degrees centigrade – check you haven't done this.

2. (a) Correct. As there are less molecules in a given area, the energy required to bring those molecules to equilibrium with their surroundings is less than for water.
   (b) Incorrect. Whether or not a substance is flammable doesn't necessarily affect its specific heat.
   (c) Incorrect. This may affect the temperature at which the molecules 'break free' from each other (the boiling point), but will not affect how much thermal energy the molecules have upon heating.

3. (a) Incorrect. You've probably taken the average temperature difference to be half of the difference between 74C and 45C, but it's the average of the two. The change in length = coefficient of expansion x length x change in temperature
   (b) Incorrect. You've probably calculated the expansion for just one degree temperature difference: you need to multiply by the range of temperature.
   (c) Incorrect. You've probably used 74C as your temperature change, (or tried to Google the answer) but this is rather an overestimate given that you were asked to take the average.
   (d) Correct: The change in length = coefficient of expansion x length x change in temperature = $1.5 \times 10^{-5} \times 1420 \times (74+45)/2$ = 1.26 km

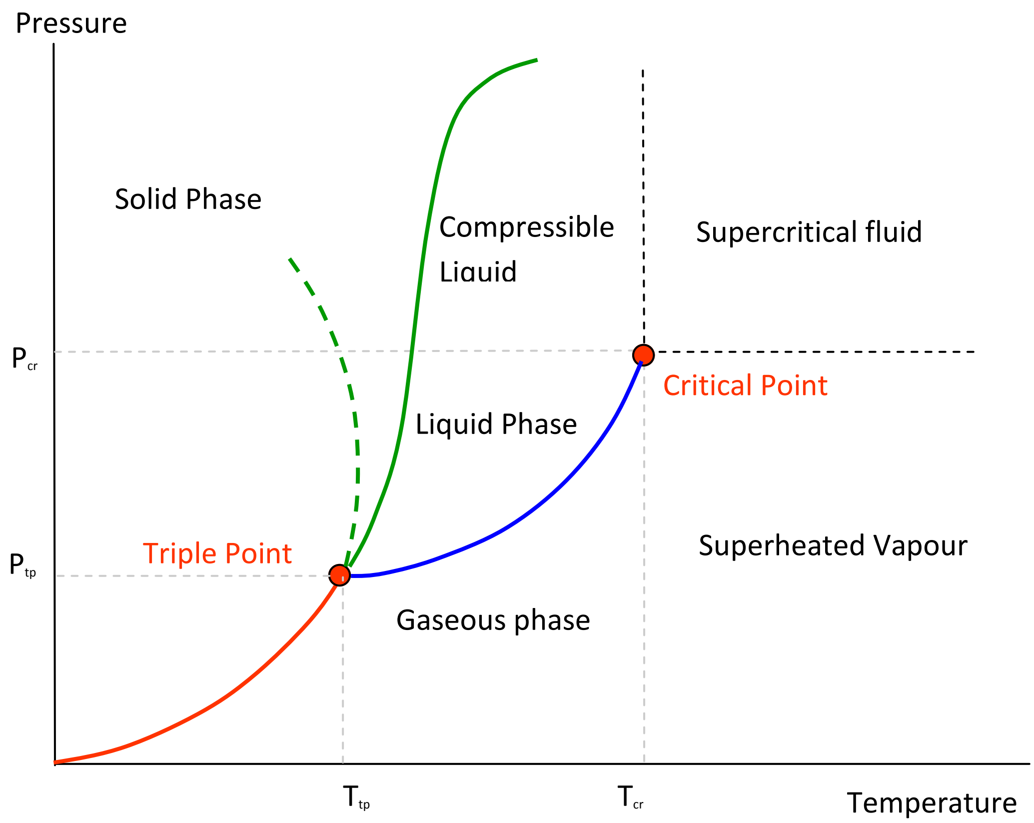
4. (a) Correct – BMR x time = Mass x specific heat x rise in temperature so 80 x 3600 = 70 x 3.49 x 1000 x T
   (b) Correct – divide energy radiated in an hour by the specific heat of the body (taking this to be mainly water)

(c) Incorrect – you probably forgot to take account of the mass of the body in estimating the heat capacity

(d) Incorrect – You probably put the specific heat as 3.49 J/kg K instead of 3.49 kJ/ kg K

Change Of State

Phase diagrams



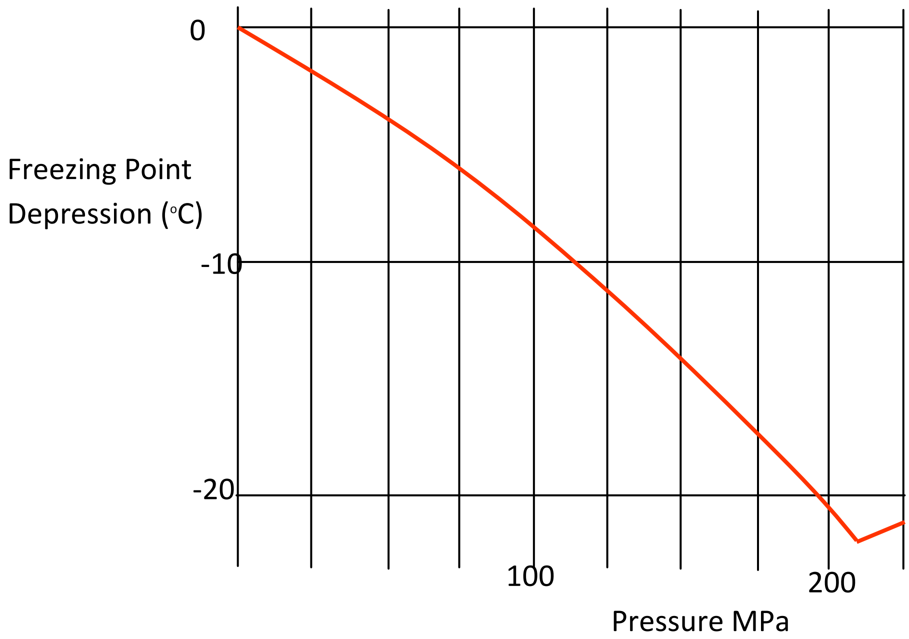
The figure shows the various phases and the boundaries between them for some typical substances on a P-T diagram. These boundaries mark the places in the diagram where the relation between pressure and temperature changes its form. Along these boundaries the two phases can coexist in equilibrium, for example water and ice at atmospheric pressure and 0^0 C. At the triple point all three phases can co-exist.

At very high temperatures and pressures, above those labelled as the critical point in the diagram, the distinction between liquid and gas breaks down. Notice that for most substances the solid-liquid phase boundary has a positive slope, given by the solid green line. In these cases the solid phase has a higher density than the liquid and pressure increases the melting point. This phase boundary for water is more like the dotted green line. As the temperature is lowered in this region, the density goes down putting up the volume. So water expands on cooling below 4^o C, and the solid has, rather unusually, a lower density than the liquid in this region. If the volume is fixed then as the temperature goes down the pressure must increase – hence we get burst pipes. Turning this round, the backward sloping curve also shows that increased pressure lowers the melting point of ice.

Pressure and Freezing Point

A favourite illustration of the effect of pressure on the melting point of water is ice skating, where the pressure of the skater's blades is supposed to raise the melting point and hence provide a thin skin of water to aid motion across the ice. There is some doubt as to whether the pressure would be sufficient to have any significant effect. Try to calculate the temperature change you would expect before revealing our estimate. Why would skating on solid carbon dioxide be a test of the theory?

By how much does the pressure under the blades of an ice skater lower the melting point? Is this a reasonable explanation of ice skating?



Answer: Pressure = weight/area

= 70 kg x 10 ms^{-2} / (1mm x 200mm)

= 3.5 MPa

The change in melting point is a fraction of a degree

Latent Heat

Let's look at some of these changes of phase in more detail. We know it takes time to melt ice or boil a kettle of water. Why is this? Is it a feature of all substances and their phase changes?

In fact, phase changes are nowadays divided into two classes: first order and second order. A phase change is first order if it involves a latent heat. In such a transition the system remains at a fixed temperature while absorbing heat. This takes time, so the system must pass through a mixed phase. You see this when water boils – bubbles of vapour mix with un-boiled water. The latent heat goes into breaking the molecular bonds in the liquid.

From the latent heat of evaporation of water the time to boil away a given amount of water from a given heat source can be calculated. We'll ask you to do this in an SAQ.

$$\text{Time to evaporate} = \frac{\text{latent heat}}{\text{rate of heating}}$$

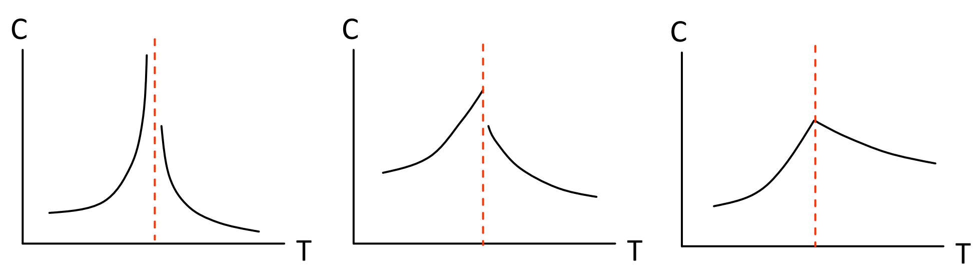
A phase change is second order if it does not involve a latent heat. The ferromagnetic transition is an example of this: there is no heat exchange with the surroundings as an iron rod is magnetised or demagnetised so the process is virtually instantaneous.

The latent heat of fusion of water at 0C = latent heat of melting of ice = 334 kJ kg^{-1}

The latent heat of boiling of water = latent heat of condensing of steam = 2260 kJ kg^{-1}

Heat capacity at a phase transition

At a phase transition various properties of a substance change their form. The figure shows the possible behaviours of the heat capacity at a phase transition.



Which corresponds to the ice-water transition?

An example of the first diagram is the ferromagnetic phase transition that we encountered in unit 7.

How do de-icers work?

We can induce phase changes by mean other than direct heating. De-icers are an example.

Image[5]

If you look up de-icers on the web you'll find that they work by lowering the freezing point of water. This can't quite be right, since they are being sprayed on to ice not water. What happens is that a little of the ice dissolves in the liquid de-icer with a release of heat. This heat is sufficient to melt some of the surrounding ice and the resulting solution of water and de-icer has a freezing point lower than the ambient temperature.

[5] JetNetherlands Dassault Falcon 2000EX PH-VBG, by Andy Mitchell UK, as posted on www.flickr.com. Creative Commons Licenced.

Equations of state

A substance that obeyed the perfect gas law would remain gaseous at all temperatures.

$PV = nRT$ *(Perfect gas equation of state)*

Real substances do not behave in this way, and therefore their equations of state must differ from a perfect gas at low temperatures or high pressures. Various attempts have been made to model this and hence to explain phase transitions. The most well-known is the van der Waals equation of state which approximates the gas-liquid transition.

$$\left(P + a\left(\frac{n}{V}\right)^2\right)(V - nb) = nRT$$

(Van der Waals Equation of state)

The parameter b is introduced to take account of the finite size of molecules and a to take account of molecular interactions in a real gas.

Modern approaches to phase changes build on the fact that the transitions do not depend on details of the molecular interactions but only on some general parameters.

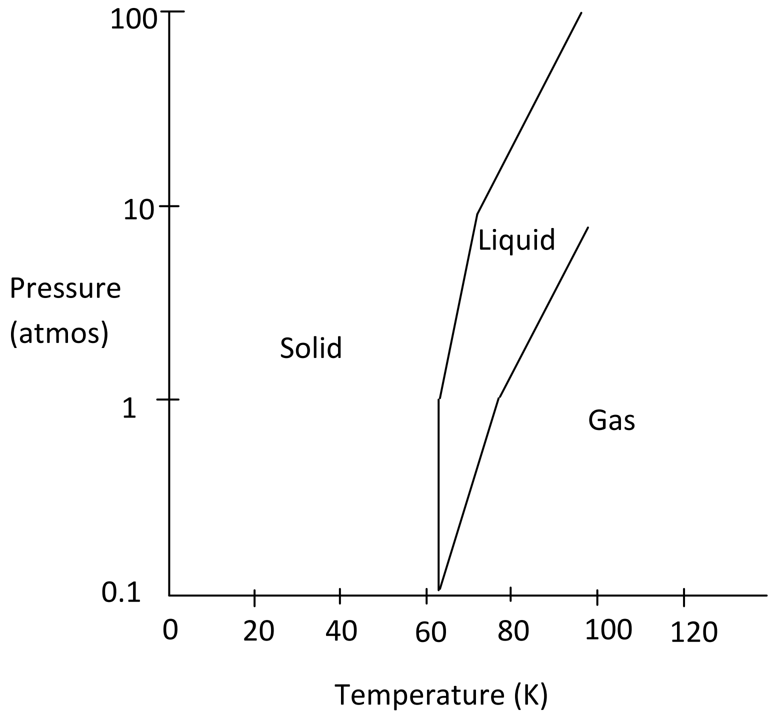
Summary

- An equation of state relates the thermodynamic variables of a system such as pressure, temperature and volume.
- The relation can be plotted on a diagram relating pairs of these variables, such as pressure and temperature.
- When this is done it is seen that the relation is not of the same form in all parts of the diagram: the different forms correspond to different phases (by definition).
- First order phase transitions involve a latent heat; second order ones do not.

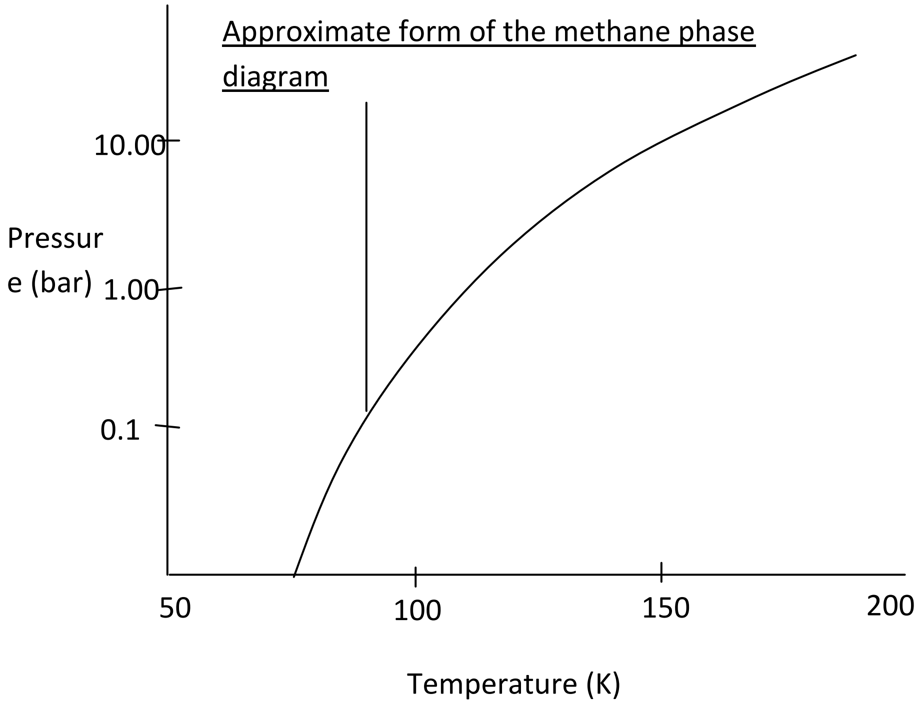
SAQs

1. Roughly how long does it take to boil away 2l of water using a 2kW heater?
   (a) 2.25 s
   (b) 38 minutes
   (c) 6 minutes

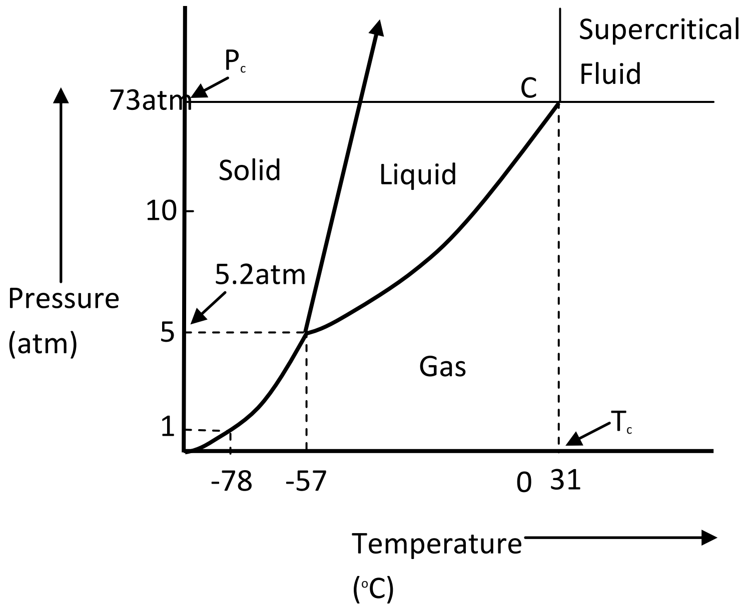
2. Triton, the largest moon of Neptune, has a tenuous nitrogen atmosphere with small amounts of methane. The surface temperature is at least 35.6 K (−237.6 °C) Triton's surface atmospheric pressure is only about 1 pascal (0.01 millibar). The southern polar region of Triton is covered by a highly reflective cap of frozen nitrogen and methane sprinkled by impact craters and openings of geysers. Why are there no nitrogen seas on Triton? *(see the phase diagram below)*
   (a) not cold enough to liquify nitrogen at this pressure
   (b) too cold for nitrogen to remain liquid
   (c) nitrogen sublimates at this pressure and temperature



3. Titan, one of the moons of Saturn has liquid methane seas. Why are there no methane icebergs?
   (a) not cold enough for solid methane
   (b) methane never solidifies
   (c) solid methane is denser than the liquid so does not float



4. From the fact that the CO_2 polar caps of Mars do not melt what can be said about the Martian atmosphere?
(a) It is very cold on Mars
(b) There is no CO_2 in the atmosphere
(c) the pressure is so low that the CO_2 sublimates



The answers appear on the following page

Answers

1. (a) Incorrect – you are probably confusing Joules and kiloJoules

   (b) Correct: The latent heat of evaporation of water is 2260 kJ kg^{-1} so the latent heat required is 2260 x 2 kJ, since a litre has a mass of 1 kg. At 2 kW or 2 kJ s^{-1} this will take 2260 s or about 38 minutes

   (c) Incorrect – you have probably used the latent heat of melting instead of the latent heat of evaporation

2. (a) Incorrect (b) Incorrect (c) Correct

3. (a) Incorrect

   (b) Incorrect

   (c) Correct – the solid-liquid boundary does not slope to the left, so pressure raises the melting point (or leaves it unchanged if the line is exactly vertical). This is the behaviour of a substance that contracts on freezing: this is the normal case in which the solid is denser than the liquid.

4. (a) Incorrect (b) Incorrect (c) Correct

Transport of Heat

Conduction

Suppose the only process acting on the iceberg were conduction; that is, assume that the melted ice remains in situ. How long would it take for the small iceberg to melt?

We'll begin with conduction in which heat is transported through a material without any mass motions in the material. We define the thermal conductivity κ (kappa) of a material by equation (1):

$$\frac{dQ}{dt} = \kappa A \frac{dT}{dz} \qquad (1)$$

The rate of flow of heat through the material is proportional to the area and to the temperature gradient. Thus, in regions with large temperature differences there will be relatively large heat flows. The equation is consistent with the fact that there is no net heat flow in equilibrium when the temperature gradients are zero.

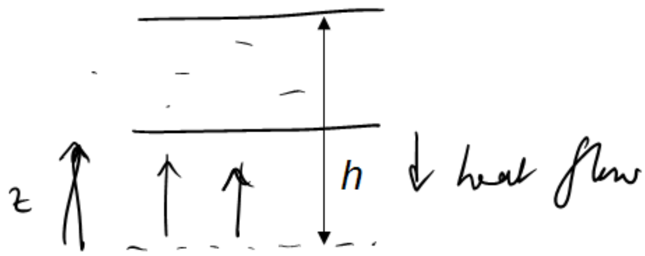
Note that there is a subtlety here. We are assigning a temperature to a material in which heat is flowing. So we are trying to use an equilibrium property, namely temperature, in a non-equilibrium situation. This can only be justified if the temperature gradients are relatively small, so that when we look at a small part of the material the temperature is practically constant over this part.

We can now look at how long an iceberg would survive if heat is being conducted to it from the sea only through the melted ice at its base. We'll assume that the water from the melting of the iceberg remains below the iceberg. Equivalently, we assume that it takes longer for the melted water to disperse than for the iceberg to melt. This is not a realistic assumption so I will give an unrealistically long lifetime.

For water $\kappa = 0.58$ W m^{-1} K^{-1}

Melting Iceberg

Consider a unit area of iceberg of height h.



We can write the time derivative of the heat flow Q in terms of the space derivative through the melted ice by the chain rule: dQ/dt = dQ/dz x dz/dt. But we know that dQ = Lρdz, because this is how much heat is absorbed on melting a strip of thickness dz. This gives us an equation for z in terms of t, equation (1) in this section which we can integrate to get equation (2). That is:

For unit area
$$\frac{dQ}{dt} = \kappa \frac{dT}{dz}$$

From the chain rule
$$\frac{dQ}{dz}\frac{dz}{dt} = \kappa \frac{\Delta T}{z}$$

But
$$\frac{dQ}{dz} = L\rho$$

So
$$zdz = \frac{\kappa \Delta T}{L\rho} dt \qquad (1)$$

Hence
$$\frac{z^2}{2} = \frac{\kappa \Delta T}{L\rho} t \qquad (2)$$

There's no constant of integration because we've taken z = 0 at t=0. We can use this equation to estimate the lifetime of our small iceberg to be about 20 years.

$$t_1 = \frac{L\rho}{2\kappa\Delta T} h^2 = \frac{334\times10^3}{2\times0.58\times5} 10^4 \left(\frac{h}{100\text{m}}\right)^2$$

$$\cong 6\times10^8 \text{ s} \qquad \text{for a 100m iceberg}$$

or about 20 years

This is an unrealistic upper limit. Note that z is proportional to $t^{1/2}$, in other words in this situation the depth of iceberg melted after a time t is proportional to $t^{1/2}$.

Convection

At the opposite extreme we can assume that the melted ice is removed as soon as possible after it is formed. The heat from the sea therefore now has to get through some layer of water (of thickness δ) that moves with the iceberg. Note that there will always be such a boundary layer because the relative speed at the solid liquid interface must be zero due to the viscosity of the water. If the layer has a thickness δ then the temperature gradient down which heat is being conducted to the ice is ΔT/ δ instead of the ΔT/z that we had previously. Making this change gives a lifetime 2δ/h shorter than before, where h is the height of the iceberg.

$$t_2 = \frac{L\rho}{\kappa\Delta T} h\delta = 2t_1 \frac{\delta}{h}$$

Note that the time is proportional to h and not $h^{1/2}$ because it now depends solely on the volume of ice to be melted rather than a diffusion time.

Estimating the thickness of the boundary layer is complicated: we've just taken 2m as typical to give a lifetime of 0.8 years. We expect the real value to lie between this and the 20 years we estimated previously.

The truth will lie somewhere in between.

The boundary layer thickness δ depends on the speed, but it is obviously considerably less than the height of the iceberg. The melting time is therefore considerably reduced.

Suppose δ is about 2 m the time becomes about 0.8 years

Radiation

However, we have not so far taken into account the fact that the Sun will also contribute to the heating of the iceberg. How much energy from the Sun falls on a unit area in a unit of time? This value is called the solar constant. We could look it up, but it's also important to see how it can be derived.

Before we turn to that, note that a blackbody by definition absorbs all the radiation falling on it. A measure of how black a body is in this sense is the so called albedo: the albedo of a body is the fraction of incident energy is reflects back. So a black body has an albedo of zero.

Solar Constant

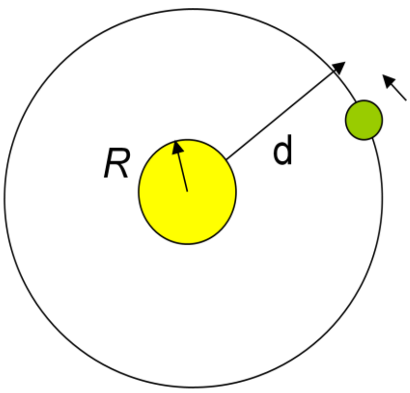
In session 11 we saw that the Sun had a Planck, or blackbody spectrum. How much energy does such a body emit? It turns out to depend on the fourth power of the temperature. The full formula is called Stephans law and it states:

Stephan's law: The energy radiated per unit time per unit area from a body of temperature T

$$= \sigma T^4$$

where σ *is the Stephan-Boltzmann constant* $\sigma = 5.670 \times 10^{-8}\ J\ K^{-4}\ m^{-2}\ s^{-1}$

Of course, this isn't what we get on Earth or the surface of the Earth would be at the temperature of the solar surface. The radiation is diluted on its way to us by the inverse square law. So, if the radius of the Sun is R and the Earth-Sun distance is d we get $(R/d)^2\ \sigma\ T^4$ per unit area per unit time at the top of the atmosphere.



We can say the flux from the Sun falling on the Earth is:

$$F = \frac{4\pi R^2 \sigma T^4}{4\pi d^2}$$

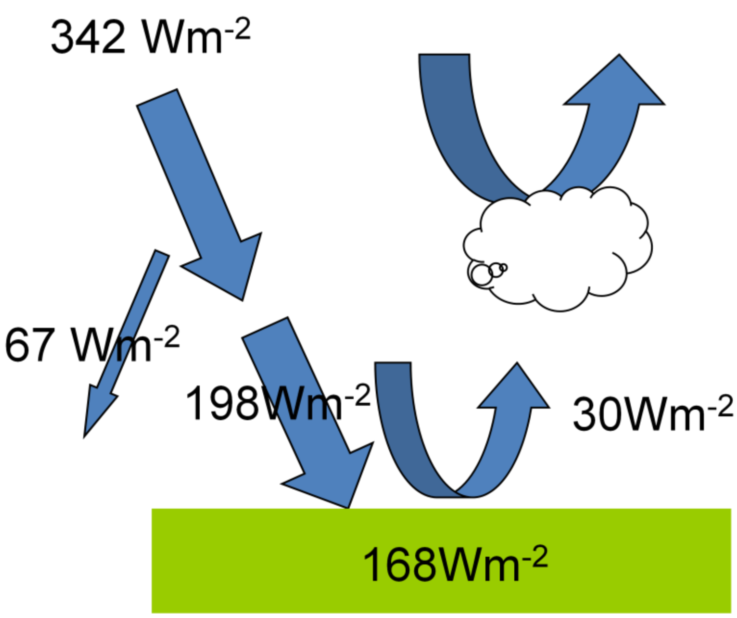
Notice that we don't need to know R and d to work this out: R/d is the semi angle subtended by the Sun in the sky which is about ¼ degree. Therefore R/d = ¼ x 2pi/360 radians or about 1/240 of a radian. If we take the temperature of the Sun to be 6000K this gives a flux of 1300Wm^{-2}. The true value of the Solar constant 1366 watts per square metre at the top of the atmosphere, though it fluctuates from 1412 Wm^{-2} in early January to 1321 Wm^{-2} in early July in the northern hemisphere.

Averaged over the Earth's surface this would be a factor 4 less. The factor 4 comes about because the solar radiation falling on the cross section of the Earth of πr^2 is redistributed over the whole $4\pi r^2$ of the Earth's surface.

In fact because of losses about 198 Wm^{-2} reaches the Earth's surface on average (over day and night) as shown in the next section.

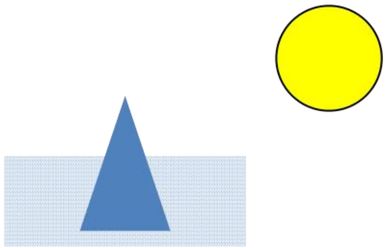
Solar Constant on the Ground

On average over a 24 hour day, each square metre of the upper regions of the atmosphere receives 342 watts of solar radiation. The atmosphere absorbs on average 67 Wm^{-2} and reflects 77 Wm^{-2} so about 198 Wm^{-2} reaches the Earth's surface. Of this 168 Wm^{-2} is absorbed and 30 Wm^{-2} is reflected back to space.



Towing Icebergs

We've looked at convection and conduction in the melting of our iceberg. We are now in a position to add the final component in our estimate of its lifetime. This is the effect of radiation.



The solar flux at the surface of the Earth is 198 W m^{-2} or about 200 Wm^{-2}. Of this the iceberg reflects back about 80% leaving 40 W m^{-2} to be absorbed. The energy goes into heating the ice to melting temperature, the latent heat of melting of the ice, and the latent heat of evaporation which will help to cool the ice. For the purpose of this estimate we've assumed that the ice is at -15^{o}C, but you can see that varying this assumption will not have a significant effect.

The calculation goes as follows:

Mass of iceberg, M = height x area x density

$= 100 \times 10^3 \times 920$

$= 9.2 \times 10^7$ kg

Ice Specific Heat: t = 2 kJ kg^{-1} K^{-1}

Heat required to warm the iceberg to 0C = $C_{ice}\Delta T\, M = 2 \times 15 \times M$ kJ

Heat required to melt the iceberg = $L_m \times M = 334M$ kJ

Heat required to evaporate the melted ice = $L_e \times M = 2260\, M$ kJ

Adding up the contributions gives a requirement of 2424 kJ per kg to melt the ice. Dividing by the energy per second absorbed by the iceberg gives a time of somewhat less than a year.

Lifetime of the iceberg = $(30 + 334 + 2260)M / FA = 2.6\ 10^7$ s < 1 year

This is close to what we found for convection. Of course, the two effects will work together to reduce the lifetime.

So our conclusion is that a small iceberg will last a matter of months before it is significantly reduced. At a towing speed of 2 m s^{-1} the iceberg could travel up to 20 000 km which is a good way round the world. It may still not be very economic of course, and it hasn't been tried.

Before going on, think about the effect of wrapping the iceberg in plastic. Then look at *our* response and see if you agree.

What would be the effect of the plastic cover recommended by Professor Quilty?

Answer: The plastic would speed up melting by about a factor of 7 because it would suppress evaporation cooling

Summary

- Stephan's law states that the flux of radiation from a black body is σT^4
- The albedo of a body is the fraction of radiation not absorbed
- The solar constant is the flux of radiation reaching the upper atmosphere
- The average flux at the surface is affected by reflection and absorption and by the curvature of the Earth

SAQs

1. Normal body temperature is 37C. The area of a typical human body (height 5′ 10′′, weight 70 kg) is about 1.87 m^2. Assuming that heat is lost solely by radiation and that a person can be treated as a black body, at an ambient temperature of 15^{o}C what approximately is the basal metabolic rate (BMR)? (i.e. the resting rate of energy loss)

   (a) 980 W

   (b) 250 W

   (c) 70 W

2. How much radiant energy falls on the flat roof of a house of area 100 m^2 each day?

   (a) 20 kJ

   (b) 1600 MJ

   (c) 2000 MJ

3. Why does it defeat the object to wipe your sweat away on a hot day?

   (a) because sweat provides an insulating layer

   (b) because sweat provides protection against UV light

   (c) because allowing the sweat to evaporate provides cooling

The answers appear on the following page

Answers

1. (a) Incorrect: You've forgotten to take account of the radiation received from the surroundings.

   (b) Correct : the net heat loss is $\sigma A(T^4 - T_0^4)$. This estimate is about a factor 3 too high which suggests that a normally clothed body is not well represented as a black body at blood temperature.

   (c) Incorrect – did you work it out or look it up? This is the correct value but does not follow from the assumptions in the question.

2. (a) Incorrect – this is the rate per second

   (b) Incorrect – you are asked for the amount of radiant energy falling on the roof so take the albedo to be 0 not 0.8

   (c) Correct – The solar constant is 200 Wm^{-2} so 200 x 100 x 10^5 = 2000 MJ

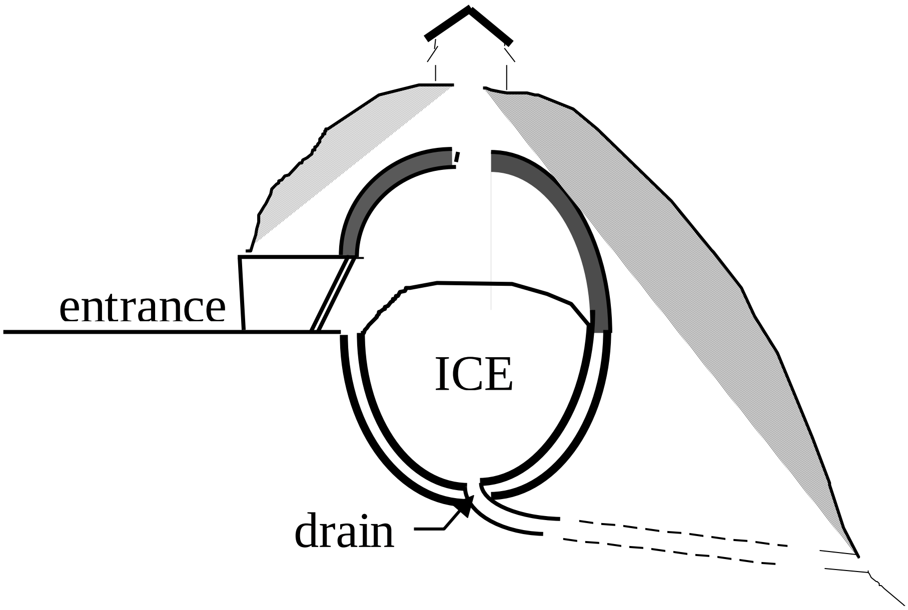
3. (a), (b) Incorrect

   (c) Correct

Additional Problems

Problem 1: Ice houses

Before the invention of the refrigerator ice gathered from frozen ponds in winter was stored in ice-houses. All large country houses had such an ice-house which provided them with a supply of ice during the rest of the year. A typical ice-house (see the diagram) was a large brick cavity-walled chamber partially or fully sunk into the ground and well insulated. The ice was broken up before loading so that it formed a compact mass inside the chamber. At the base of the chamber there was a drain hole to take away the melt water which would otherwise accumulate in the bottom of chamber and spoil the heat insulation provided by the straw bundles which lined the chamber's inner surface.



Assuming that heat is conducted into the ice through the cavity wall and straw lining with which it is in contact, and that no ice is taken out of the ice-house, our problem is to estimate the half-life (the time for half the ice to disappear) of this ice-house from the following information. The measured temperature at the surface of the ice is about 3^{o} C

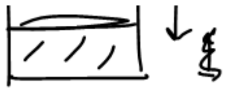
We'll take the density of ice in the chamber to be 800 kg m-3 and the temperature of the ground to be 8 ^{o}C.

The heat conduction rate through the cavity wall and straw lining is 1 $Wm^{-2}K^{-1}$. Note that this is not the same as the thermal conductivity we met in section 3. In section 3 we multiplied the conductivity by the area and temperature gradient to get the rate of loss of heat. Here, the thickness and composition of the walls has been taken into account in the conduction rate: we multiply the conduction rate by the area and temperature difference to get the rate of loss of heat.

Answer:

To solve this problem begin by letting $\dot{M}$ be the rate at which the mass of ice diminishes by both evaporation and melting. Let L be the weighted latent heat of evaporation and melting – that is the actual latent heat supplied to melt and evaporate the ice taking into account if necessary any loss of liquid water from the ice house that does not evaporate. Then the total energy loss is $\dot{E}$ =L $\dot{M}$.

$\dot{M}$ = mass loss to evap. and melting. A



$L\dot{M}$ = heat loss rate

$$\dot{E} = -L\dot{M}$$

$$A = 2\pi r h = \frac{2M}{\rho r}$$

$$M = \pi r^2 h \rho$$

This energy is lost by conduction through the walls where the ice is in contact. We'll ignore losses through the floor of the ice house. So $\dot{E}$ is the heat conduction rate K times the area in contact times the temperature difference. The area depends on the mass of ice left. Assuming the icehouse is cylindrical with radius r, the area of a cylindrical mass M with density ρ is just twice M divided by r times ρ. Putting $\dot{E}$ in as – L $\dot{M}$ and doing a little rearrangement gives us equation (1) with $\lambda = 2K\Delta T/L\rho r$.

$$\dot{E} = K\Delta T A$$ Ignore floor and consider only walls.

$$: K\Delta T \frac{2M}{\rho r} = -L\dot{M}; \quad \dot{M} = -\lambda M \quad (1)$$

This is just the radioactive decay law so we know the halflife is $\log_e 2/\lambda$. We've worked this out in two cases.

$$\therefore \text{half-life} = \frac{\log_e 2}{\lambda} = \frac{\log_e 2}{2u\Delta T} \cdot \text{yr}$$

$$\sim 3 \times 10^7 \text{ s}, \quad \text{if } L = L_M$$

$$\sim 21 \text{ yr} \quad \text{if } L = L_E$$

So ~ 2 yr if $\frac{1}{7}$ g ice evaporates.

First when there is no evaporation – this gives about 3 times 10^7 seconds or about a year. Second when there is ONLY evaporation – this is unrealistic, but it approximates the case when all the ice that melts also evaporates and no liquid water is lost. The lifetime comes out as a very large 20 years. In practice, unless the icehouse is airtight, there will be some evaporation cooling, so the lifetime will lie between the two extremes. Since the latent heat of evaporation is 7 times that of melting the two contributions will be equal if 1/7 of the ice evaporates (and the rest flows away as liquid). In this case the lifetime will be extended by a factor 2 over no evaporation.

Why is it preferred to allow the ice to be exposed to the air?

Problem 2: Sooty icebergs

One suggestion for dispersing icebergs from shipping lanes was to cover them with soot in the expectation that this would speed up melting. By how much would the lifetime be altered by this approach?

Answer: Without soot the albedo of ice is around 0.8. With soot it is 0. So the use of soot in theory reduces the lifetime by a factor of 5. (100% is absorbed to melt the ice rather than 20%) The sooty water will continue to absorb around 80% as long as the soot is not washed away.

Overall Summary

- The zeroth law of thermodynamics allows us to define temperature
- Bodies at the same temperature are in thermal equilibrium with no heat flows
- The specific heat of a body is the heat input per unit rise in temperature
- The latent heat of a body is the heat required to change its phase
- A second order phase transformation involves no latent heat
- An equation of state is a relation between the thermodynamic properties of a body
- Heat may be transported by conduction, convection or radiation
- The conductivity of a material is the rate of flow of heat per unit area per unit temperature gradient
- The coefficient of thermal expansion is the relative change in size per unit change in temperature
- The Stephan-Boltzmann law states that a black body at temperature T radiates energy at a rate σT^4 per unit area.

Meta tags

Author: Cheryl Hurkett.

Owner: University of Leicester

Title: Enhancing Physics Knowledge for Teaching – Heat

Keywords: Thermodynamics; Equilibrium; Change of state; sfsoer; ukoer

Description: In this session we'll begin our study of thermodynamics by looking at some of the properties of heat.

Language: English

Version: 1.0

Additional Information

This pack is the Version 1.0 release of the module. Additional information can be obtained by contacting the Centre for Interdisciplinary Science at the University of Leicester. http://www.le.ac.uk/iscience