Part 1 Particles and Forces

Session 1 Mechanics

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Contents

Welcome

Welcome to session 1 of the Physics programme. The approach we'll be taking in this session will set the structure for the whole of the module. We'll begin by introducing a problem that will cover the main learning objectives of the session. We'll then look at what is required to solve this problem; We'll build up this knowledge step by step, applying it to the solution of the problem as we proceed. When we get to the end we will have found a solution to the problem. Then we'll invite you to try some problems covering again some of the topics that have arisen during the session, either on your own or with guidance. We'll also invite you to raise any issues with these problems in the tutorial.

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Learning Objectives

- Define velocity and acceleration
- Use constant acceleration formulae
- Define equilibrium as the balance of forces
- Use Newtonian equations of motion to compute the behaviour of bodies subject to unbalanced forces
- Use graphical representation and explain the advantages of so doing
- Demonstrate a knowledge of conservation of energy and momentum and their use in applications
- Solve problems involving work and power
- Use dimensional analysis to derive simple relationships

Section 1: Forces

Problem 1: Making Lead Shot

The lead shot used in shotgun cartridges consists of small spherical pellets 2-3mm in diameter and this is made by pouring molten lead through a frame suspended in a high tower, a method that has been used since its invention by William Watts in 1782. Now in order to produce spherical shot the lead has to solidify before the pellet has reached its terminal velocity. How high should we build the tower?

A shot tower at Redcliff, Bristol.
Photo by [Yellow Book Ltd]

Let's see what we need to find out about this problem: we'll call these our learning issues.

You might like to stop and think about this before going on.

This is my list so far:

We need to find out what is meant by terminal velocity and why it is important?

We need to think about how we can find the distance fallen through to reach terminal velocity?

What happens before terminal velocity is reached?

What happens afterwards that prevents the formation of spherical shot pellets?

Let's get back to basics: what causes the motion of the shot? – clearly the forces acting on it.

So one of our first questions will be: What are the forces on the shot?

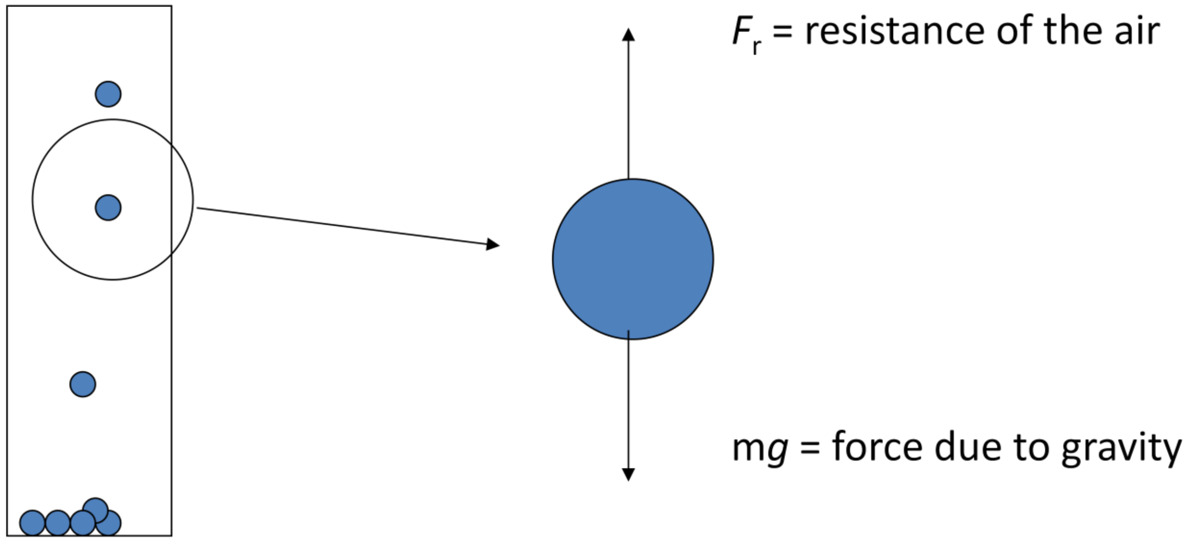
Equilibrium: Balance of forces

Interactivity: Drop a ball (or any other object you have to hand) and describe what you actually see as it falls.

University Students almost always claim to see what they've been told happens – namely that the ball falls with constant acceleration. If you claimed this is what you saw, do it again. You can't possibly tell that this is motion under constant acceleration, at least not with the unaided eye. To a first approximation it looks as if something odd happens to start with as you release the ball, and then the speed remains pretty constant after that. (Why shouldn't it? – after all, there's nothing in the way to do any pushing or pulling - or so one might think). This is a fundamental issue. Research shows that many university students understand less about physics after a course in mechanics than they did before it! What students learn to do is to make the correct responses in the classroom (without believing them) and carry on with their naive Aristotlean perceptions outside the classroom. This makes physics totally irrelevant to these students.

The problem with seeing what is happening here is the strength of gravity – 9.8 m /sec^2 is a big acceleration for the unaided eye to follow. To solve this problem, we can effectively weaken gravity by watching motion on an inclined plane, as Galileo discovered. We'll come back to this in the next session. For the present, let's take it that acting alone gravity produces a constant acceleration.

The gravitational force on a body of mass m is its weight, mg.   The other force on the body is the resistance of the air.



Balance of forces => equilibrium => constant speed

Imbalance of forces => acceleration

The figure shows how we can divide the fall of the shot into two phases. If forces of gravity and air resistance balance we have equilibrium. You might think that equilibrium entails an absence of motion. In fact, in equilibrium there can be motion but there can be no acceleration. So at this stage the shot is at its terminal speed. If the forces on a body don't balance the body will accelerate. This is what happens up until the shot approaches terminal speed.

So we know the gravitational force on the shot.

Returning to the problem: how do we find the force of the airflow on the shot, which we've called F_r?

Dimensional Analysis

To get the dependance of this force on the relevant physical quantities we can try an approach known as dimensional analysis. We start by identifying what the relevant physical quantities are for the falling shot pellet. You might like to stop and suggest your own.

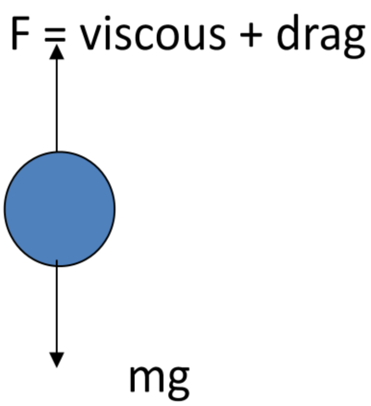
We identify the radius of the shot *a*, its speed *v*, the density of medium (air) ρ, and the viscosity η as possible relevant quantities here.

This gives us a problem. Dimensional analysis works, as we'll see in a moment, by balancing dimensions of mass, length and time on both sides of an equation. That's three things to balance – three equations – which can't possibly determine four physical quantities'

What we have to do is to guess that under some circumstances one might not matter. We guess that the speed must come in as must the radius under any circumstances. Suppose however that the viscosity is very very tiny; can the body experience a resistance to motion? Obviously, if it's moving fast enough it will still encounter resistance from moving the air in front of it out of the way. So there is a regime where the viscosity doesn't matter. In this case we can show that the Force is proportional to $A\rho v^2$.

Note that this result is not an approximation – the functional dependance is exact, only an overall constant is missing. In this case it's a constant that depends on the shape of the body – usually it's around 0.25. We'll use this when we come to make accurate calculations later. For the moment we'll ignore the constant. Alternatively we might think of circumstances where the viscosity is probably important (think of treacle) and the density less so – this gives Stokes' formula. If we let the force depend on viscosity we would get $F \propto a\eta v$. You might like to try to derive this by dimensional analysis. In this case the missing constant is a rather more significant 6π, but that can only be determined by a much more sophisticated mathematical treatment.

One mistake students always make is to assume that all forces have to be included under all circumstances – sort of just in case approach. It's true they are all always there, but there's only a narrow regime in which all can have a discernable effect if they depend on different physical quantities. So the next thing to do is to see which of the three forces due to gravity, viscosity and air resistance, are the important ones for the falling shot.



Initially **v** is small. Then $v^2 < v$, so initially the relevant force is proportional to v and the drag can be neglected.

We have gravity minus the viscous term causing acceleration

Estimate viscous term:

$6\pi a\eta v = 6\pi \times 0.003 \times 1.5 \times 10^{-5} v \sim 10^{-6} v$ N

But $mg = 11\,000 \times (4/3\, \pi a^3) \times 10 \sim 10^{-2}$ N

Key fact: the SI units (and their abbreviations) are

mass: kilograms (kg); length: metres (m); time: seconds (s)

Wikipedia gives density of lead = 11.34 gm cm^{-3} We're using SI units, so how do we convert this into kg m^{-3}? – don't use a formula. We know 1kg is 1000 g so 11.34 g/cm^3 is 11.34/1000 kg

m^{-3}. And $1m^3$ is 100x100x100 = $10^6 cm^3$, so 11.34/1000 kg m^{-3} is 11.34/1000 x 10^6 kg m^{-3} or 11340 so say 11000 in round numbers for our estimate. Note that we don't know what speed the pellets have, so we've left it is the formula as v.

By comparing the two values in equation (1) we see that at speeds less than 10^4 m /s the viscosity term is negligible compared to gravity. – so we don't include it! Hence for the initial fall of the shot we have constant acceleration under gravity

So our Next Learning issue will be: how do we compute the behaviour of the shot under const acceleration? First some revision.

Summary

- Force of Gravity = *mg*
- Unbalanced forces cause acceleration
- Use of dimensional analysis
- Comparison of forces to determine which are important

In this section we've used the fact that the gravitational force on a mass m is mg. We've seen that unbalaced forces on a body result in accelerations. We've seen how to use dimensional analysis to derive the force due to air resistance. And we've seen that in making a mathematical model we neglect any effects that are insignificant: often this can lead us to break up the motion of a body into different phases.

SAQs

1. In terms of mass, length and time, what are the dimensions of (i) energy (½ mv^2), (ii) pressure (Force/area)

   (i) (a) MLT^{-1} (b) ML^2T^2 (c) ML^2T^{-2}

   (ii) (a) MLT^{-2} (b) $ML^{-1}T^{-2}$ (c) ML^3T^{-2}

2. What are the SI units of (i) energy (½mv^2), (ii) pressure (Force/area)

   (i) (a) Joules (b) Newton metres (c) Watt seconds

   (ii) (a) Newton metre^{-2} (b) Pascal (c) Joules (metre)$^{-3}$

3. What is 90 mph in ms^{-1}? (Use 1km = 5/8 mile) (enter only the numerical value without units)

   (a) 0.04 (b) 1.5625 (c) 40

4. The diffraction of a beam of light by a circular aperture produces a spot that has a size depending on the wavelength of light, the radius of the aperture and the distance between the screen and the aperture. Despite the fact that the problem involves just three parameters, dimensional analysis cannot be used to determine the size of the spot. Why not? (No knowledge of the behaviour of light is required to answer this question.)

   (a) Because the dimensions involved are all lengths so the answer will have an unknown dependence on the dimensionless ratios of the lengths.

(b) Because the formula involves addition

   (c) Because this is not a problem in mechanics

   (d) Because the formula involves an unknown constant.

The answers are on the following page.

SAQ Feedback

1. (i) (a) the velocity has to be squared (b) Velocity is LT^{-1} so $(velocity)^2$ is L^2T^{-2} (c) correct – the dimensions are $Mx(LT^{-1})^2 = ML^2 T^{-2}$

   (ii) (a) MLT^{-2} are the dimensions of force, not pressure (b) correct: pressure is force/area = mass x acceleration/area = $(MxLT^{-2})/L^2$ (c) this is force x area not force/area

2. (i) (a) Joules, Correct (b) Literally correct, but not the SI name for the unit (c) Literally correct, but not the SI name for the unit

   (ii) (a) Literally correct, but not the SI name for the unit (b) Correct (c) Literally correct, but not the SI name for the unit. However, for order of magnitude estimates it is very often useful to recall that pressure is the same as energy per unit volume.

3. (a) you probably forgot to convert km to m (b) 1 mile is 8/5 km so you should multiply by 8/5 not 5/8. (c) correct. 90 mph = 90x8/5 km h^{-1} = 90x8/5x1/60x1/60 m s^{-1}

4. For this question:

   a. correct: if a formula involves a dimensionless ratio it cannot be derived by dimensional analysis

   b. It is true that formulae involving addition of two terms cannot be obtained by dimensional analysis, but that is not the issue here

   c. The fact that the fundamental units are mass, length and time, does not restrict the use of dimensional analysis to mechanics problems. All quantities can be expressed in these units.

   d. Multiplicative constants can never be found by dimensional analysis: all that can be determined is the proportionalities.

Section 2: Kinematics

Kinematics deals with the description of motion independent of its causes. So to study the initial motion of the shot we need to deal with the kinematics of constant acceleration.

We should start by defining speed. Well, you know that Speed = distance/time. Stop and think a moment. This is not obvious – you've already forbidden your students to add unlike quantities (what does speed + time = ?) so why are they allowed to divide them? (Again, one rule for the physics class another for real life?) Speed is the distance travelled in a unit of time – dividing is the way to get this, which is why it's valid.

Everyone learns the constant acceleration formulae so I won't try to persuade you not to bother. But at least let's not learn them by rote - Let's see why they are obvious.

First we have to define our terms: u is the initial speed, v is the final speed after a time t and a distance s under a constant acceleration a.

Then the equation (1)

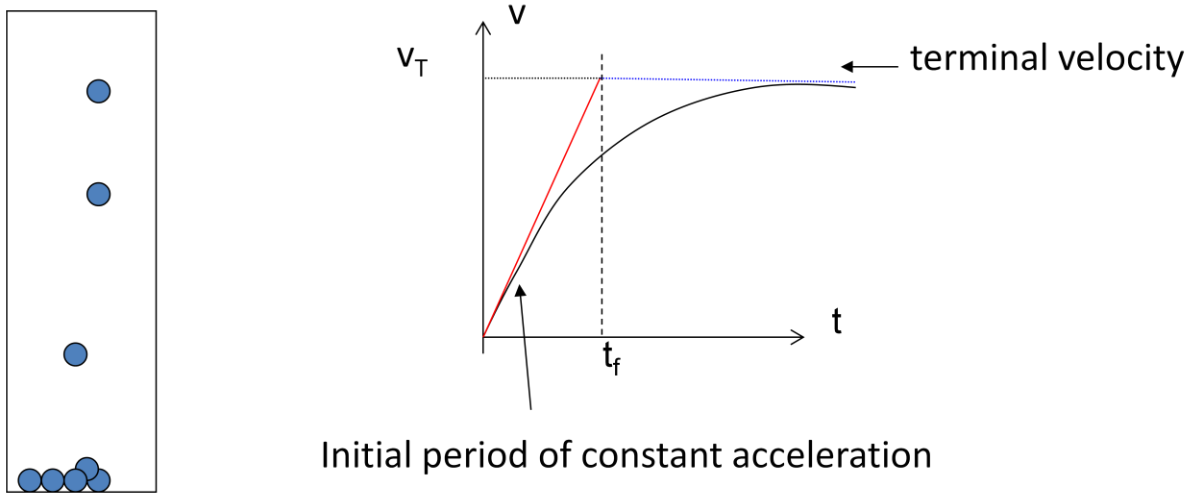
$v = u + at$ – is the definition of acceleration – as what is added to the speed per unit time

In the equation (2)

$s = ut + 1/2at^2$ – the ut is obvious – it's where you'd be at constant speed. The at^2 comes from at (extra speed) x t (time) – why the ½ - because the extra speed isn't *at* – it's on average ½ as much.

$v^2 = u^2 + 2as$ This is where maths comes in! I have no idea how to make this formula obvious (without using ideas extraneous to kinematics). We know it is true because it can be derived from the other formulae by algebra. (Do it.) So we turn this round: the maths gives us a new insight into how the speed depends on the distance a body is accelerated for – its quadratic in v. (or equivalently, the speed increases from rest with the *square root* of the distance.)

Interactivity: Sketch how you think the velocity of a shot pellet (y-axis) depends on time (x-axis)



Followed by descent at the 'terminal' velocity

Now before we solve the problem mathematically we want to look at what we expect to happen. One way to do this is to sketch a graph. At this stage we don't know the most useful graph to draw – speed versus time, speed versus height , or some other combination, so we might experiment with a few possible choices. The simplest to think about is speed versus time. Later we'll see that this is not the most convenient, but let's look at it for the present. The shot starts off with constant acceleration so moves off in a straight line ($v = at$) on this graph. That's what makes this graph simple – we immediately see that a non uniform acceleration implies a curved line and vice versa. Note that we're measuring distance from the top of the tower downwards, so the shot has a positive velocity even though it's falling (its distance from the origin is increasing). Actually, on the graph we're plotting speed rather than velocity, so this is positive anyway.

The shot eventually reaches a terminal speed so the line must curve over to become parallel to the time axis. These two sections are joined by some smooth curve, which at this stage we can't calculate but we can sketch roughly.

The approximately constant acceleration phase (the red curve) intersects the approximately constant velocity phase (the blue line) around a time t_f on the graph. This time therefore divides the two phases of approximately constant acceleration and approximately constant speed.

We can now make an order of magnitude estimate of the quantities involved algebraically.

Algebraic Estimates

The constant acceleration phase comes to an end when air resistance balances gravity, equation (1). Solving this for v gives us the terminal speed, equation (2).

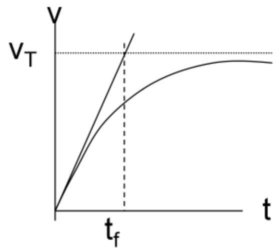
To find the time t_f and the corresponding height of drop H, we use the constant acceleration formulae. Equation (3) gives the time and equation (4) the height.

How long does the constant acceleration phase last?
What is the minimum height of the tower?

Drag balances gravity when $\pi a^2 \rho v^2 = mg$ (1)

This relation gives the terminal speed

$v = v_T = \sqrt{(mg/ \pi a^2 \rho)}$ (2)



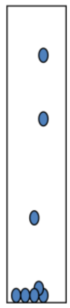
To find how long and what distance this takes use constant acceleration formulae:

Check dimensions:

(3) $v = gt$ implies $t = t_f = v_T/g = \sqrt{(m/\pi a^2 \rho g)}$ $(M/L^2ML^{-3}LT^{-2})^{1/2} = T$

and

(4) $v^2 = 2gh$ implies $h = H = v_T^2/2g = m/(2\pi a^2 \rho)$ $M/(L^2ML^{-3}) = L$



What happens thereafter? – const $v = v_T$

Note how in the right hand column we check the dimensions of our formulae to ensure we haven't made a mistake. Note also how when we're working out an algebraic result we use symbols for ALL the quantities, even the ones that have known constant values. This enables us both to check dimensions and to check for other mistakes more easily than if we had carried through a string of numerical values. But the main advantage is that it clearly displays the dependence of any result on the input data – for example the dependence of the time on the air density ρ.

Note also that we go back to $v^2 = 2gh$ to get the height rather than $s = ½gt^2$. Don't use intermediate results if possible, because that way mistakes don't propagate through a calculation

Once the forces of gravity and air resistance balance the acceleration is zero, so they continue to balance. The speed therefore remains constant at the terminal speed, which is of course the reason for then name

Can we make some numerical estimates from this to answer our original problem? Let's estimate the time taken to fall first.

$$t = \sqrt{(m/\pi a^2 \rho g)} = \sqrt{(\rho_{lead}/\rho_{air})(a/g)}$$

1 $\rho_{lead} = 1100\ kg\ m^{-3}$

$\rho_{air} = 1.29\ kg\ m^{-3}$ $t = 0.50578053885887315051809360572175\ s$

$a = 0.003\ m$

$g = 10 m\ s^{-2}$

2 $\rho_{lead} = 1134\ kg\ m^{-3}$

$\rho_{air} = 1.29\ kg\ m^{-3}$ $t = 0.518\ s$

$a = 0.00300\ m$

$g = 9.81\ m\ s^{-2}$

Key fact: NEVER EVER quote answers to more dps than the data

Interactivity: work out H to an appropriate precision. Check that your answer is reasonable

1 has the greater number of decimal places so is the more precise; But it can't be more accurate than the 1 dp of the input data. 2 has the greater accuracy because the data is given to greater precision. Results should never be quoted with more dps than the data.

Our result is as precise as the data allow, but is it accurate? We've only given an approximation. Is this the best we can do? – Can we find an exact relation between v and distance fallen? -

Our next learning issue will be to see how forces are causing motion to get a more accurate description of the motion. This takes us into dynamics. First some revision.

Summary

In this section we looked at

- Constant acceleration formulae
- Graphical analysis
- Terminal velocity or speed
- Accuracy and precision

SAQs

1. Match the quantities in the set (a), (b), (c) to those in the set (i), (ii), (iii) by balancing the dimensions (c = speed of light, G = Newton constant Nm^2kg^{-2})

   (a) the lifetime *t* of a black hole of mass *M* and radius *R*

   (b) the energy per second *L* radiated in gravitational radiation by a body of mass M in an orbit of radius R

   (c) the escape speed from a planet of mass *M* and radius *R*

   (i) $M^3c^6G^4$

   (ii) $G^{1/2}M^{1/2}/R^{1/2}$

   (iii) $G^4 M^5 / c^5 R^5$

2. If a sprinter who clocks 10 s for the 100m were to be accelerating at a constant rate, how fast in ms^{-1} would he be traveling across the finish line? [Put in a number in numerically form with no units]

The answers are on the following page.

SAQ Answers

(a) – (i), (b) – (iii), (c) – (ii)

20 [m s^{-1}]

Section 3: Dynamics

In this section we shall begin by deriving the Newtonian equations of motion for a point particle. The details of the derivation itself are not as important here as the general idea. Once we have the equation of motion we can apply them to our problem to get a more accurate representation of the motion, which will confirm our previous estimates.

Deducing the conservation of energy

In standard university courses we normally think of deducing the conservation of energy from Newton's laws of motion. This is not how it's done in fundamental physics (including string theory!) we start from energy, so that's what we'll do here. [In more complex situations with many *df* we need special techniques to extract the equations of motion for each df, but the principle is the same]

Think up any number of coordinates to describe a system – let's stick to one (x) for our example. Think up an expression for energy involving x. You can make up (an infinite number of) your own theories of dynamics at this point, but sticking with the one that works we take (1).

$$E = \tfrac{1}{2} m v^2 + U(x)$$

or

$$E = \tfrac{1}{2} m \dot{x}^2 + U(x) \qquad (1)$$

The first term is the energy of the free system – ½ mv² or ½ $\dot{x}^2$ - (without any interactions); the dot here dates back to Newton himself and is used to indicate a rate of change or derivative wrt (with respect to) time.

The second term in equation (1) gives the interactions of the system with the world. In this case, when the world only makes an appearance through parameters in U, we call U a potential energy. Later we'll use PE to introduce the notion of a field. We'll also be able to clarify the nature of various types of energy.

Here we just mention a useful example: in a gravitational field a body of mass m raised through a height x has PE mgx Most students remember this as mgh but we use x here, not h to indicate that it is a variable coordinate of the system, not a fixed value.

Where are Newton's first and third laws in this picture? – The 1st law is NOT obtained by putting U or F=0. It says that frames of reference exist in which this derivation holds. i.e. with respect to $\ddot{x}$ which is the measured acceleration. These will be the so called inertial frames, which are considered to be non-accelerating so that we can use them as standards wrt which to measure accelerations.

The third law only emerges here if we have more than one body: then it is contained in the symmetry of U with respect to the coordinates of those bodies. (And if U is not symmetric the third law is not satisfied.)

Now derive Newtonian mechanics from conservation of energy. In this simple case of one coordinate we just differentiate the energy equation wrt time. The derivative of ½ m $\dot{x}^2$ is $\dot{x}$ x $\ddot{x}$. To get the derivative of U wrt time, we use the chain rule: U changes only because x changes so its the rate of change with respect to time is the rate of change wrt x, dU/dx, times the rate of change of x wrt t, dx/dt or $\dot{x}$. Cancelling $\dot{x}$ and defining dU/dx to be the force F gives the Newtonian equations of motion (3).

$$\frac{dE}{dt} = m\dot{x}\ddot{x} + \frac{dU}{dt} = m\dot{x}\ddot{x} + \frac{dU}{dx}\frac{dx}{dt} = m\dot{x}\ddot{x} + \frac{dU}{dx}\dot{x} = 0 \qquad (2)$$

So

$$m\ddot{x} = -\frac{dU}{dx} = F \qquad (3)$$

Note: you can go back the other way if you like. Reverse the steps to see how energy conservation can be derived from Newtonian dynamics for a force that can be expressed as the derivative of a potential energy.

Many problems in dynamics can be solved using conservation of energy without needing Newton's 2nd law (These are before and after problems) We'll see this later in the course. Let's try to apply what we've learnt here to the shot problem.

Equations of motion for the shot

We immediately see a difficulty. Newton's laws are derived assuming conservation of energy, but with air resistance this isn't true. We have to assume that Newton's 2nd law also applies to the force of friction, such as air resistance. This is an assumption. To prove it we should have to investigate the nature of friction which we are not going to do here. So lets assume that we can just add in the frictional force F_r (with the correct sign) to mg to get the total force, equation (1)

$$m\ddot{x} = F = mg - F_r \qquad (1)$$

Now we have a choice of how we solve this equation, corresponding to the choices we had of which graph to plot – v against time or distance and so on. We'll choose to present v as a function of distance of fall, simply because we've tried the alternatives, and this give the clearest picture. .

To start with then we need an expression for acceleration that involves v and x to put into (1). We have $d^2x/dt^2 = dv/dt = dv/dx \times dx/dt$ by the chain rule, which can be written as $v\,dv/dx$ as in equation (2). For those of you who are interested, lets see how we then solve the equation of motion for $v(x)$.

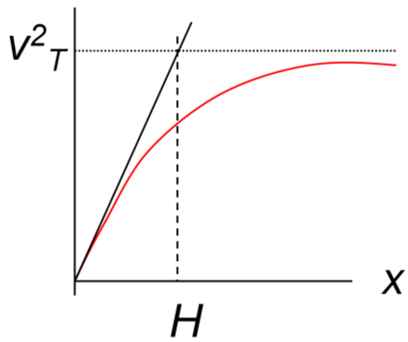
$$(2)\quad m\frac{d^2x}{dt^2} = m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt} = \quad mv\frac{dv}{dx} = mg - \pi a^2 \rho v^2$$

$$\text{to get} \quad v^2 = \frac{mg}{\pi a^2 \rho}\left[1 - \exp\left(-\frac{2\pi a^2 \rho x}{m}\right)\right]$$

$$= v_T{}^2[1 - \exp(-x/H)] \qquad (3)$$

for the speed of the shot as a function of distance x from the top

The solution is shown both as equation (3) and as the graph. Note that the graph is not $v(x)$ or $v(t)$ – In fact we've plotted v^2 as a function of x, distance not time. PICTURE is NOT $v(x)$!!!



This tells us the shot never reaches its exact terminal speed! But for x large enough, the speed is near enough the terminal one. If we take x about $m/2\pi a^2 \rho$

as in our previous estimate, then we are about $(1-e^{-1})^{1/2}$ of the terminal speed so only about 40% out way there.

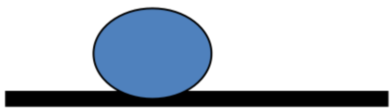
Nevertheless this gives us confidence in our approximate approach to provide the correct order of magnitude.

The shape of the pellets

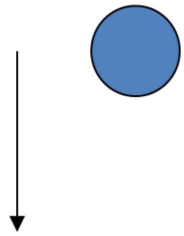
The original problem asked us to determine the shape of the shot pellets.

The simplest approach is to put ourselves in the position of the falling shot – that is we transform ourselves to the reference frame of the shot.

Once the shot has reached terminal speed we put ourselves in its frame of reference by a constant velocities transformation. In this reference frame the shot is at rest with the force of gravity balanced by the air resistance - it's just as if the pellet were resting on a table. Therefore, if it's still liquid the drop will spreads out – it is not spherical!



constant velocity



free fall

Consider next the initial stages of its fall, while air resistance is negligible and only gravity is acting. The shot is in free fall so in its frame of reference it is weightless. In these conditions the liquid pellet will adopt a spherical shape under the action of surface tension.

Incidentally the shape of a falling raindrop is not spherical for just this reason.

Summary

- In this section we have studied the derivation of Newton's laws from an energy function:
- Their extension to cases where friction is present, the application to the shot problem using:

$$\frac{d^2x}{dt^2} = v\frac{dv}{dx}$$

- And the shape of the shot in free fall and at terminal speed.

SAQs

1. How do the constant acceleration formulae relate to Newton's 2nd law?

   (a) They are logically independent because they describe kinematics, not dynamics

   (b) They are equivalent in the special case of a constant force

   (c) Constant acceleration is not possible in practice

2. At what point do astronauts become weightless?

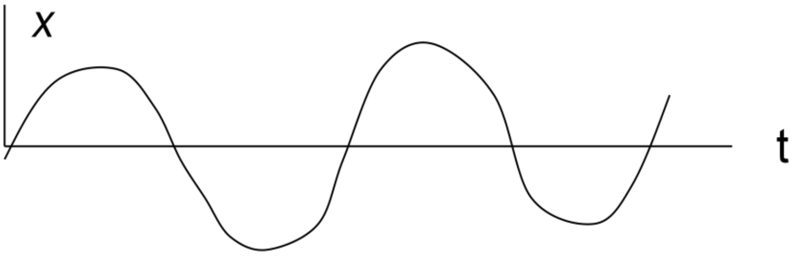
   (a) In orbit

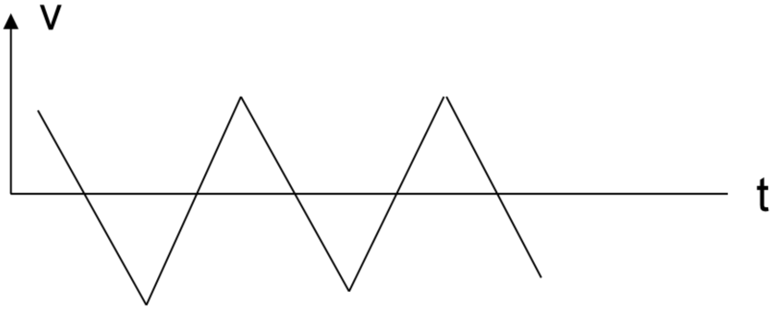
   (b) As soon as the rocket motors are switched off

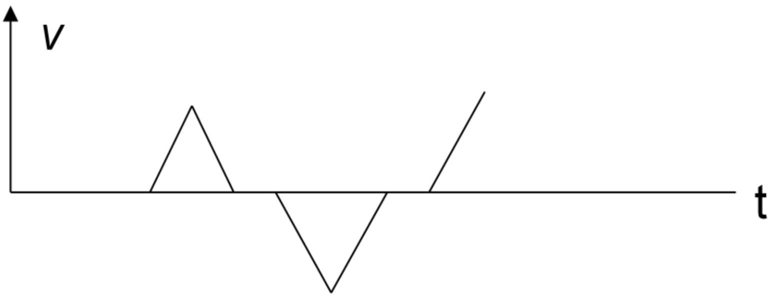
   (c) as soon as they leave the ground

   (d) never

3. Match the equation of motion $\frac{d^2x}{dt^2} = -x$ to a displacement-time graph.

(a)


(b)


(c)


4. Arthur C Clarke suggested that high g-forces could be reduced to zero by immersing a body in a fluid at neutral buoyancy. Is this true? YES/NO

The answers are on the following page.

SAQ Answers

1. (a) dynamics tells us what forces are necessary to set up kinematic relations – in particular how constant acceleration can be achieved (via a constant force)

(b) correct: the law tells us that we need a constant force to maintain a constant acceleration

(c) But it's near enough that we can model the behaviour of real bodies in this way

2. (a) they are weightless in orbit but this is not their first experience

(b) Correct – once they are in free fall i.e. there are no non-gravitational forces

(c) At this point they experience high g as the rocket accelerates as well as Earth gravity

(d) once they are in free fall i.e. there are no non-gravitational forces they will experience weightlessness

3. (a) Is correct: the acceleration (curvature of the velocity graph) increases as the distance from the origin increases (i.e as the velocity decreases) and is the velocity is then reversed

(b) I guess you think the spikes come from the reversal of the motion when the deceleration has brought the mass to rest. But the reversal of the velocity must occur when it is at zero – which is quite smooth

(c) I guess you're thinking that the velocity hovers around 0 as it reverses; but the acceleration isn't zero when the velocity is zero – it's zero when there is no displacement (x =0).

4. NO: The set-up is exactly the same as the falling of lead shot at its terminal speed when viewed in the rest frame of the shot or the body. Here, the body in rest frame is suspended in water instead of air. There are non-zero forces on the body which balance; the force on the body is not zero. A similar well known problem is the forces on a book on a table. It is not true that there are no forces because the book has zero acceleration. The book has zero acceleration because the forces balance.

Section 4: Conservation Laws

In this section we'll look briefly at work done by a force and the conservation of energy. We'll also look at the conservation of momentum and see how these concepts apply to the shot problem.

The energy of falling shot

In the free-fall phase the only force on the shot is gravity

In this phase the total energy is a constant: $E = \frac{1}{2} m\dot{x}^2 + U(x)$ = constant

The Kinetic Energy increases at the expense of Potential Energy.

For a system and its environment energy is always conserved.

Sometimes it is convenient to represent the effect of the environment by friction

In this case the energy of the system is not conserved, but Energy loss = work done = Force x distance (or integral)

We can see this for the shot in equation (1)

$$\frac{d}{dx}\left(\tfrac{1}{2} mv^2 + mgx\right) = F_r \quad \text{so} \quad \tfrac{1}{2} mv^2 - mgx = \int_x^0 F_r dx \qquad (1)$$

Energy loss = Work done

Energy non-conservation can also occur if the model has an explicitly time dependent energy – but that is a situation also obtained only by leaving out part of the system (the cause of the time variation).

Conservation of Momentum

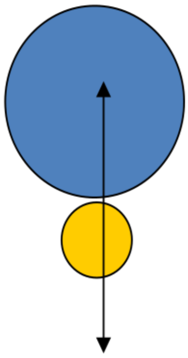
The momentum of a body is the producd mass times velocity. Unlike energy therefore, which has only a magnitude, momentum has both magnitude and direction. Note that the force on a body is its rate of change of momentum. This is easy to prove for a constant mass (equation 1), but it's also true for varying mass.

$$\frac{d}{dt}(mv) = m\frac{d^2x}{dt^2} = F \quad (1)$$

If no external forces of any nature act on a system its momentum is conserved. In this case the potential energy can depend only on the internal relative coordinates of the system and not on the position on space of the system. This is easy to see: if the potential energy U were to depend on the position of the system, x, then dU/dx would be the force acting on the system and, since a force would be acting, momentum could not be conserved.

In the terminal phase the shot is acted on by both gravity and friction. How does this lead to a steady speed?

Let's look in a bit more detail on the interaction between the air and the shot. The interaction involves millions of collisions between the shot and the air molecules. Each collision transfers momentum from the shot to an air molecule. Newton's third law ensures that in these collisions the force on the shot = force on air molecule; so the rate of change of momentum of shot = rate of change of momentum of air. Note that the air molecules then act as intermediates in an exchange of momentum between the shot and the Earth. Friction here arises from fluctuations in momentum as a result of a large number of collisions. In fact, frictional forces are always associated with fluctuations – a lot of puzzling paradoxes can be constructed by forgetting this.



3rd law

Summary of our solution to the shot problem

The shot falls under the dominant influence of gravity with constant acceleration until air resistance becomes significant.

Once air resistance is significant the shot reaches a terminal speed in which resistance and gravity balance. We can find the order of magnitude of the force due to air resistance by dimensional analysis.

We can either estimate when the end of the constant acceleration phase occurs or we can solve the equations of motion exactly.

We can work out the shape of the shot by examining the forces in the rest frame of the shot. Once terminal velocity is reached the shot cannot remain spherical. This the tower need be no higher that that required to achieve terminal velocity.

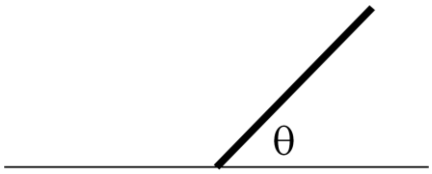
This gives $\boldsymbol{H = m/(2\pi a^2\rho)}$ for a pellet of mass **m**, radius **a** and air density **ρ**. *No 5* shot has radius 1.4mm which gives $H = 48.5$ m.

The energy and momentum lost by the shot is taken up by the air molecules.

SAQs

1. The body of mass m in the picture has a gravitational PE of

(a) **mghcos(θ),** (b) **mgh** (c) **mghsin(θ)**



2. Can a perfectly elastic body suffer damage in a collision?

Further problems with outline solutions

Dimensional analysis

How does the period of a pendulum depend on its length?

What properties of the pendulum and its environment are essential to the situation?

mass of the pendulum m

length of the pendulum l

gravity g

How do you think the period will change when we change the parameters?

Foucault's Pendulum.
Photo by Ben Owtrowsky.

By equating powers of time length and mass you should find:

$$\alpha = 0,\ \beta+\gamma=0,\ \gamma=-\tfrac{1}{2} \text{ so } T \propto (l / g)^{1/2}$$

Why is this an amazing result?

Discussion is on the following page.

These problems illustrate some of the points you have learnt.

It's always a good idea to think about outcome of a problem before tackling it: we know that longer bobs swing more slowly – so T depends on l directly. Less gravity – less acceleration – longer period – So T depends on g inversely. Perhaps also more massive – greater gravity – so perhaps T depends on m directly. T might also depend on composition – a gold pendulum might swing differently from an iron one of the same mass, but we can't put that into a dimensional analysis, so we'll leave it aside till lesson 19.

We can solve this by dimensional analysis in the same way as we did for the air resistance. Begin by letting the period depend on $m^{\alpha} l^{\beta} g^{\gamma}$.

Note: Appropriate dependencies for g and l. (although you can't guess the square root). Note also can't get l dependence on its own (need g as well)

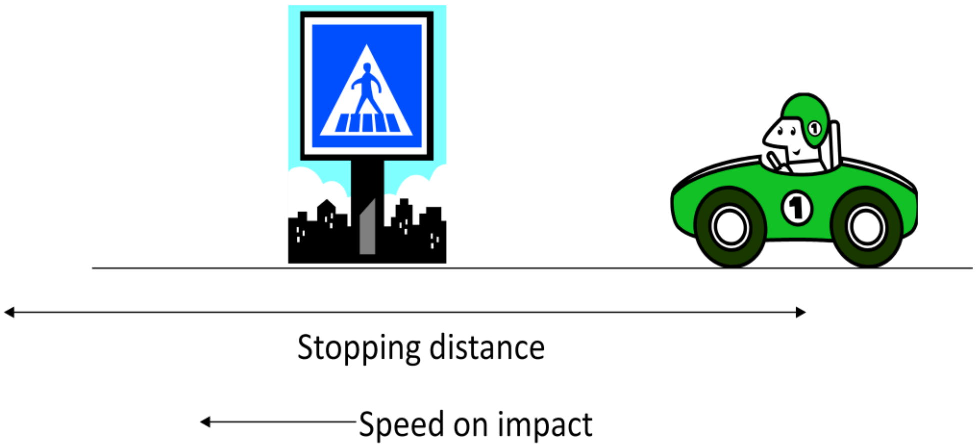
But also an AMAZING result: period is independent of m. Where did we go wrong? Greater gravity, but also greater inertia! They cancel out. Why? – it took Einstein to see this as a problem and find a solution – in General Relativity (we'll return to this in session 19)

Note why dimensional analysis works – we can identify one length (the size of the bob, the thickness of the string, the angle of swing etc are considered irrelevant.) So stripping the system down to its simplest components means we can make quantitative predictions.

Constant Acceleration

A Dept of Transport information film shows how much the stopping distance of a car travelling at 35 mph is increased over one travelling at 30 mph for the same deceleration. It might have more impact to show the speed of impact instead. Suppose a car can just brake from 30 mph to stop short of a pedestrian. What would be the speed at impact in braking from 35mph?

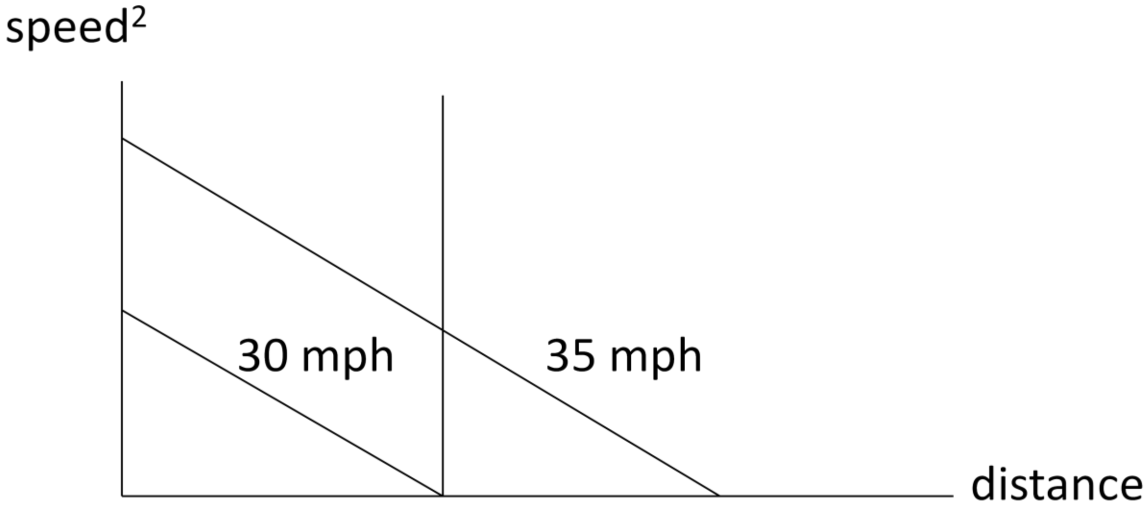
Would this make people aware of the dangers of speed?



The answer isn't 5 mph! That's non-linearity for you!

How do we go about solving this. You can use the data for stopping from 30 mph to get the deceleration in terms of the fixed distance. If you know the deceleration you can work out the speed at impact from the new initial speed over the same distance. Incidentally, it's much easier if you work the problem out in symbols and only put values in at the end.

Discussion is on the following page.



Why are the graphs straight lines?

What difference would the inclusion of a reaction time make?

The graph shows that the quadratic dependence on v is the reason for the high speed of impact – about 18 mph

Notice how the graph illuminates the reason why the impact is at such a high speed.

You can also use this to demonstrate the effect of reaction times – add a reaction time before you start to brake and see what a difference it makes. This is why you don't use a mobile phone while driving.

Power and Energy

Drag racers accelerate from rest at constant power over a straight course and the one with the highest terminal velocity wins. How does the terminal speed depend on the power of the car?

Key fact: acceleration at constant power is not the same as constant acceleration

$$\text{Power} = \frac{d}{dt}(\tfrac{1}{2}mv^2) = v\frac{d}{ds}(\tfrac{1}{2}mv^2)$$

$$\text{Thus } \boldsymbol{power \propto v^3}$$

This is a problem about the conversion of energy (the chemical energy in the fuel) to kinetic energy. Think about how you would tackle the problem before proceeding.

The power of the car is constant – that's what all the wheel spin at the start is about. The power is the rate of change of energy, hence the rate of change of 1/2mv^2. Since it's the distance that determines the end of the race, we want the rate of change with distance, not time: hence Power = v (Δ(1/2mv^2)/ Δs) So for fixed Δs, v^3 is proportional to P. (Note you don't have to integrate the equation, because we don't need the constant of proportionality.)

Conservation of Momentum

Criminals prefer light weight guns because they are more easily concealed. Why are they in more danger of hurting themselves when firing such a gun?

Key fact: momentum is always conserved

energy is conserved in elastic collisions

What do we need to know: It's a before and after question, so we need conservation laws. Conserve momentum

to see that the recoil speed is inversely proportional to the mass of the gun. The energy of course depends on MV^2 so is also inversely proportional to the mass of the gun.

Conserve momentum: $mv = MV$ So the speed of recoil is $V = (m/M)v$

You have a similar issue in firing a shotgun – it's essential to rest the gun on your shoulder before firing it.

Summary

Section 1 **Dimensional analysis:** the functional dependence in some systems can be analysed dimensionally.

Section 2 **Kinematics:** constant acceleration formulae;

$$v = dx/dt,$$
$$a = d^2x/dt^2 = v dv/dx$$

Section 3 **Dynamics** Equations of motion are obtained from energy conservation (by differentiation)

$$\text{Force} = \text{rate of change of momentum} = ma$$

Section 4 **Conservation laws** Momentum is conserved if there are no external forces; Energy is always conserved, but can be shared with the environment

Section 5 **Further problems**

Meta tags

Author: Derek Raine.

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Title: Enhancing Physics Knowledge for Teaching – Mechanics

Keywords: Forces; Kinematics; Dynamics; Conservation Laws; sfsoer; ukoer

Description: The approach we'll be taking in this session will set the structure for the whole of the module. We'll begin by introducing a problem that will cover the main learning objectives of the session. We'll then look at what is required to solve this problem; We'll build up this knowledge step by step, applying it to the solution of the problem as we proceed. When we get to the end we will have found a solution to the problem. Then we'll invite you to try some problems covering again some of the topics that have arisen during the session, either on your own or with guidance. We'll also invite you to raise any issues with these problems in the tutorial.

Language: English

Version: 1.0

Additional Information

This pack is the Version 1.0 release of the module. Additional information can be obtained by contacting the Centre for Interdisciplinary Science at the University of Leicester.
http://www.le.ac.uk/iscience