

Session 9

Geometrical Optics

Samuel Atarah

Physics Innovations Centre
for Excellence in Learning and Teaching

π CETL (Leicester)

Department of Physics and Astronomy

University of Leicester

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Welcome

Welcome to session 9 of the course. This section is concerned with Optics; we shall be interested in the behaviour of visible light as it interacts with plane and curved surfaces.

Session Author

Samuel Atarah, University of Leicester.

Session Editor – Tim Puchtler

Learning Objectives

- Give an account of reflection at and refraction at a plane boundary including Snell's law
- Give an account of reflection and refraction at curved boundaries
- Derive the lens makers' formula for paraxial rays
- Show knowledge of how lenses work and their use in examples of optical instruments
- Discuss total internal reflection

The Problem

The problem involves vision correction. Reports of the earliest use of visual aid are those of the Roman statesman, Seneca who is said to have read text through a glass globe of water. This predates even the use of reading stones which were also hemispherical transparent objects placed on top of text for magnification. Today, many people wear glasses to correct their sight. Glass globes, reading stones and lenses are all vision correcting objects that function similarly. How do they work?



Image.¹

Let's identify some learning issues. *How does an image form?*

We need to know what happens to a ray of light as it passes from one medium to another, e.g. from air to glass, and glass to air.

How does the shape of the interfacial boundary between the different media, planar or curved, affect the final ray?

What features of a glass (globe or hemisphere) make it useful for vision correction?

We shall start from a simple example of image formation – the reflection of light.

¹ Spectacles, bongo vongo, as posted on www.flickr.com. Creative Commons Licensed

Reflection

We consider a daily observation that will help us to understand and be able to trace the path of a light ray. We start with polished surfaces and then go on to consider general plane and curved surfaces.

The picture on this page is familiar to most of us from the bath room. Our interest in optics is to find out how this works. How does one picture arise from the other and how are the two pictures related in size or orientation?

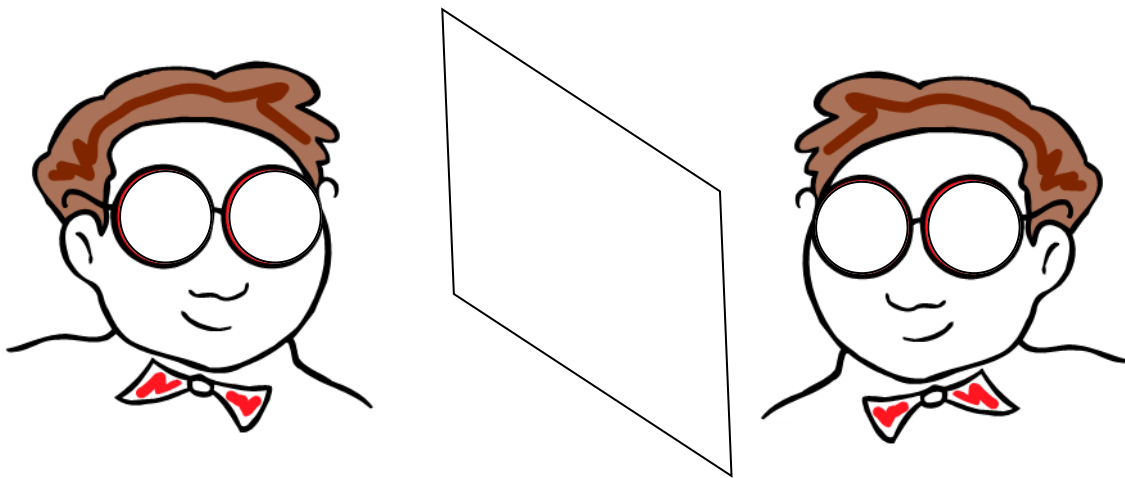
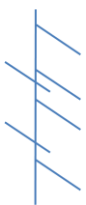


Image adapted from Microsoft Clipart.

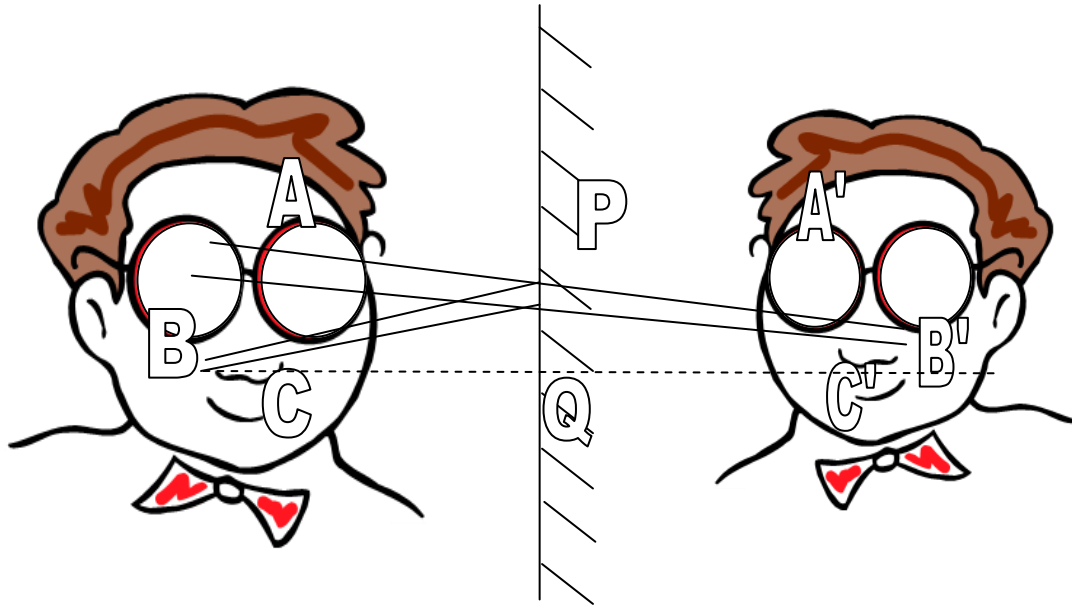
Diagrammatically, a mirror is represented by a 'comb' line with the back of the comb representing the polished surface.



Mirror symbol

Properties of Reflection

Of the two pictures, the one on the polished side of the mirror is called the **object** and the other, arising from it is called the **image**. Conventionally the object is often on the left side and the image forms on the right.



The triangles BQP and B'QP are congruent, so $BQ = B'Q$

The section shows a small bundle of rays that enter the eye that originate from the object at B, but are traced back apparently from the image at B'. Since the angle of reflection equals the angle of incidence the triangles BQP and B'QP are similar, and since QP is a common side, they are congruent. Thus, the image distance QB' equals the object distance BQ .

Because the image cannot be formed on a **screen** it is said to be **virtual**. The image is right-left inverted relative to the object. Incidentally, many people think that this inversion does not make any difference in looking at their face in the mirror, because faces are symmetric. In fact, most faces are surprisingly unsymmetric: it can be quite amusing to demonstrate this by cutting a picture of a face in half along a vertical axis and then reflecting this half to make up a full face. The result looks unlike the original face.

Angle of incidence = angle of reflection

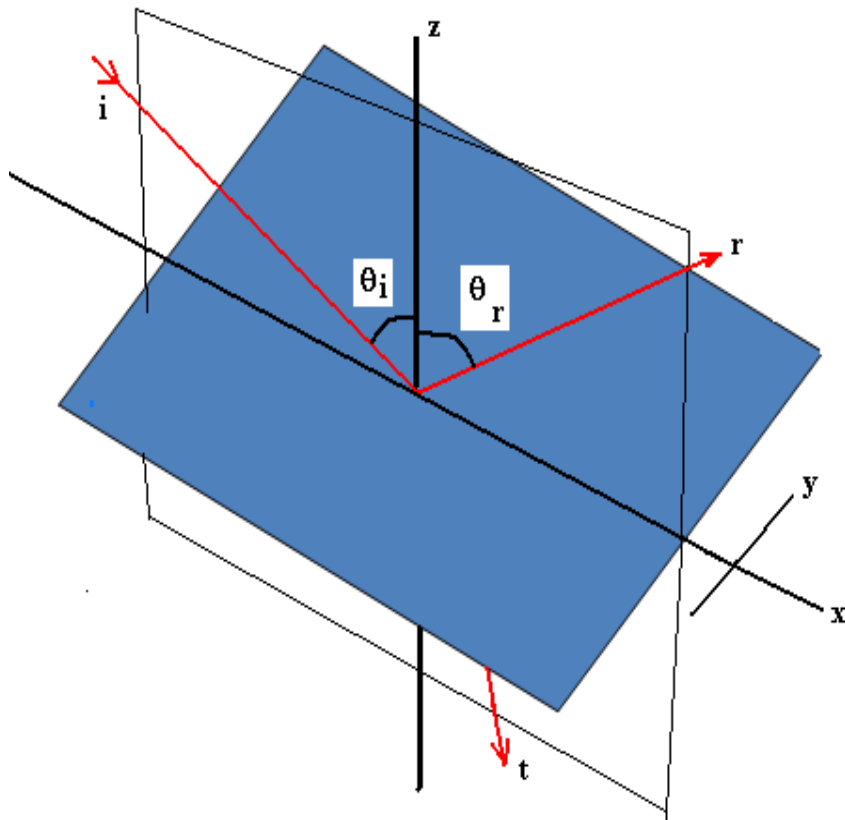
Image distance = object distance

Mirror images are virtual, and depth inverted, $R \rightarrow L$

Magnification is unity

Reflection in 3D

The preceding section makes one assumption that we bring out explicitly here, namely that the normal, incident and reflected rays all lie in a plane and that plane is perpendicular to the interface.

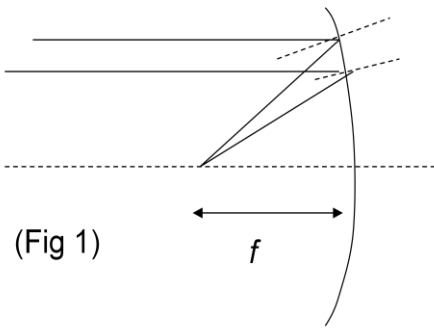


Laws of reflection

- $\theta_i = \theta_r$
- The **incident**, (i), **reflected**, (r), and **transmitted**, (t), rays all lie on same plane; the plane containing the normal and incident.

Reflection on Curved Surfaces

Consider now reflection from a curved surface. Figure (1) shows parallel rays from infinity being brought together at a point of the axis: this is the focus. Parallel rays will be brought to a focus only if the mirror has an appropriate shape. Remember that the angles of incidence and reflection have to be equal for each ray. Figure (2) shows what happens in general – for an arbitrary curved surface parallel rays are not brought to a focus. Although we shall not prove this, the shape of mirror required to bring parallel rays to a focus is a paraboloid – a parabola rotated about its axis. However, provided the shape is close enough to a paraboloid we shall get approximate focussing. This will be the case for rays sufficiently close to the axis, hence for sufficiently small apertures.



(Fig 1)

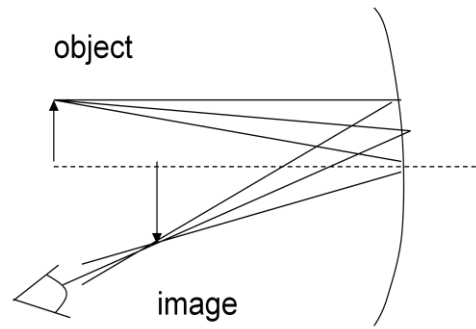
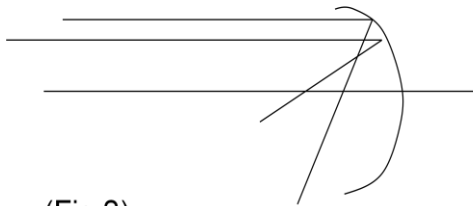


Fig 3



(Fig 2)

Why is the existence of a focus important? This is shown in figure 3. The corollary of the fact that parallel rays from a point at infinity come to a focus is that non-parallel rays from the same point on an object cross at a single point to form an image. A bundle of these rays entering the eye are traced back to the image point. For a non-parabolic surface the rays all cross at different points and there is no image, although rays close to the axis will cross at nearby points and an approximate image will be formed. Note that in this case the image is real – it would appear on a screen placed in the image plane.

Note that bundles of rays coming from different parts of the object enter the eye from the corresponding points on the image – so we see the whole object.

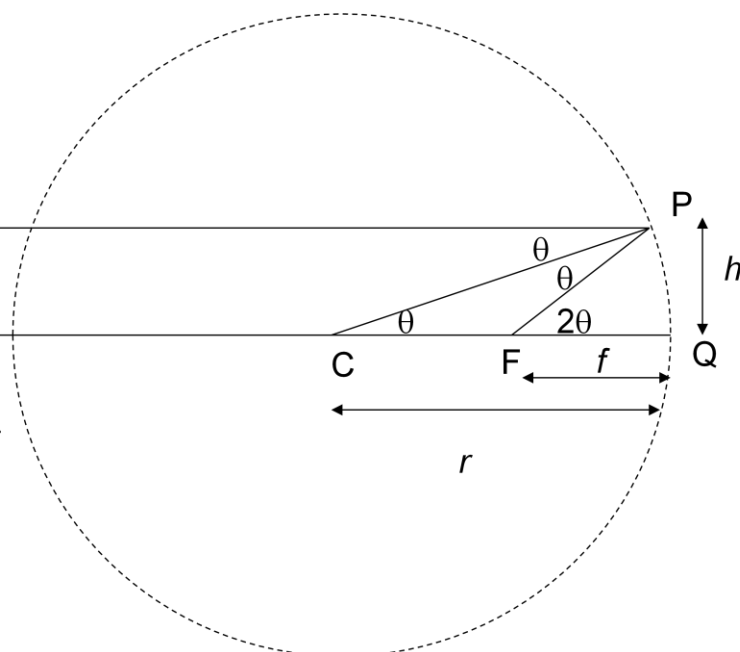
Focal Length

The focal length of a mirror is related to its radius of curvature – the more highly curved the mirror the shorter we would expect its focal length to be. The figure shows that the focal length of a mirror is in fact half the radius of curvature. The angle θ is the angle of incidence and reflection and the angle PFQ is 2θ because it is the external angle of the triangle CFP . h is the curved length of the mirror to the point of incidence. From triangle CPQ : $\theta = h/r$ and

From triangle CPQ: $\theta = h/r$

From triangle FPQ: $2\theta = h/f$

Therefore $f = r/2$

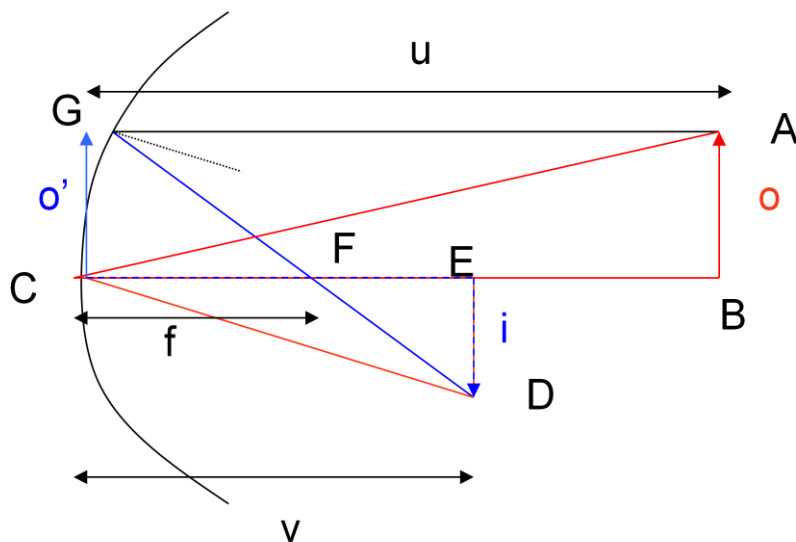


from triangle FPQ: $2\theta = h/f$. from which we can readily deduce the result, that $f = r/2$.

For rays close to the axis (or for a parabolic mirror) the focal length is independent of where the ray strikes the mirror

Focal Length from a curved reflector

Look now at the formation of an image i from the object o by a curved reflector. The object distance from the mirror is u and the image distance is v . We are going to find the relation between these two distances. To form the image take two rays AGD and ACD.



The first of these passes through the focus at F, the second through the centre of the mirror at C. We need one construction: move the object o to C where it is labelled as o'. We also need one approximation: that o' joins C and G. The figure shows that this is clearly an approximation since if the base of o' is at C, its tip cannot be exactly at G.

We now look at the four triangles containing o, o' and i. Triangles ABC and DEC are similar i.e. they have the same angles, so the ratios of corresponding sides is the same. This gives us that $i/o = v/u$ – which tells us the magnification of the object.

Next we look at triangles EDF and CGF: These again are similar because they have the same angles, so this time

$i/o' = (v-f)/f$. But, of course, i/o' is the same as i/o .

From triangle ACB and DEC $i/o = v/u$ (1) Magnification equation
Small angle approx:

From EDF and CGF: $i/o' = i/o = (v - f) / f$ (2) Focus equation

From (1) and (2):
Key fact: $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ (3)

Thus equating the two expressions for i/o gives us a relation for u , v and f . Dividing through by v and tidying things up gives us the 'lens maker's formula', equation (3). Remember that this is only an approximation for rays close to the axis, or equivalently for small angles of incidence on the mirror, even for parabolic mirrors.

Summary

Properties of Reflection:

- The object and the image are **equidistant** from the mirror line.
- The angle of reflection is equal to the angle of incidence, both angles taken as the angle between the ray and the normal.
- Mirror images are **virtual**, and **inverted, R→L**
- The magnification (size of image:size of object) is **unity** for images formed from reflection by a plane mirror
- The incident, reflected, and transmitted rays and the normal to the mirror all lie in a plane.
- For curved mirrors, the object distance u , image distance v and the radius of curvature are related by $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{r}$

SAQs

1. A ray is incident on a mirror making an angle θ with the surface.
i) What is the angle of incidence? (ii) What is the angle of reflection?
(a) θ (b) $\pi/2 - \theta$ (c) $\pi - \theta$
2. A mirror has a radius of curvature of 50cm. What is its focal length?
(a) 100 cm (b) 50 cm (c) 25 cm
3. A mirror of radius of curvature 90 cm is used to form an image of a distant star. How far from the mirror should the screen be placed to bring the image into focus?
(a) 180 cm (b) 90 cm (c) 45 cm

The answers appear on the following page

Answers

1. (i) (a) Incorrect: the angle of incidence is **always** measured from the normal not the surface
(b) Correct: the angle from the normal is $90^\circ - \theta$ or $\pi/2 - \theta$ if θ is measured in radians
(c) Incorrect: the angle from the normal is $90^\circ - \theta$ or $\pi/2 - \theta$ if θ is measured in radians
1. (ii) (a) Incorrect: the angle of incidence and reflection are the same and both are measured from the normal
(b) correct: the angle of incidence equals the angle of reflection
(c) Incorrect: the angle of incidence and reflection are the same and both are measured from the normal
2. (a) Incorrect: the focal length must be less than the radius of curvature – in fact it's $r/2$.
(b) Incorrect: the focal length must be less than the radius of curvature – in fact it's $r/2$.
(c) Correct: $f=r/2$
3. (a) Incorrect: the screen must be placed at the focus which is at $r/2$, or 45 cm
(b) Incorrect: the screen must be placed at the focus which is at $r/2$, or 45 cm
(c) Correct: the image of an object at infinity will be formed at the focus at $r/2 = 45$ cm

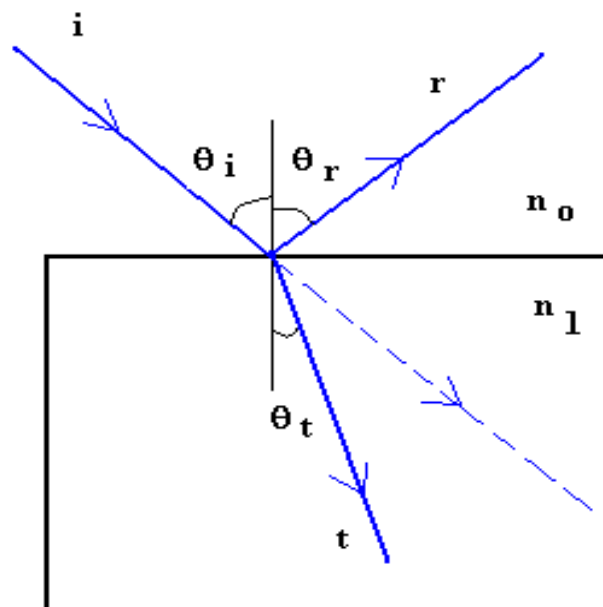
Refraction

One of the learning issues in the reading glass problem was to understand what happens as light travels from one medium to another of different density. Let's consider first the case for which there is a planar surface separating both media.

Refraction at plane surface

Recall that light travels more slowly in a dense medium than in air or in vacuum. The ratio of speeds is given by the refractive index of the medium. This has the consequence that in going from a less dense medium to a denser one, a ray of light deviates towards the normal making the angle on the transmission side, **the angle of refraction**, smaller than the angle of incidence.

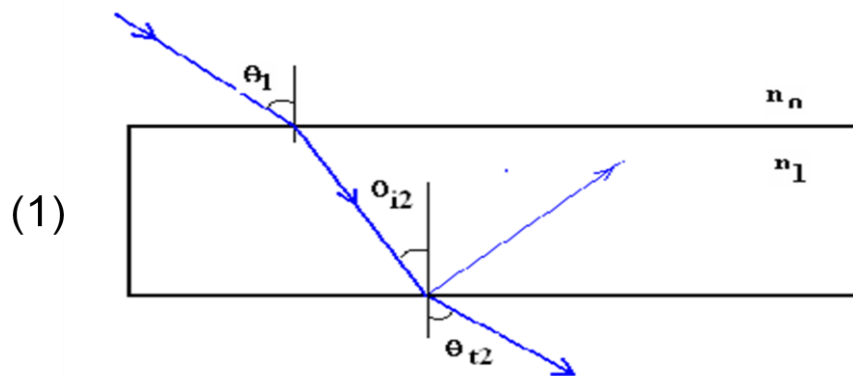
This behaviour is summed up by Snell's law, $n_0 \sin \theta_i = n_1 \sin \theta_t$ where θ_i and θ_t are the angles of incidence and of refraction respectively and n_0 and n_1 are the refractive indices of the medium on the incident and the refracting side respectively.



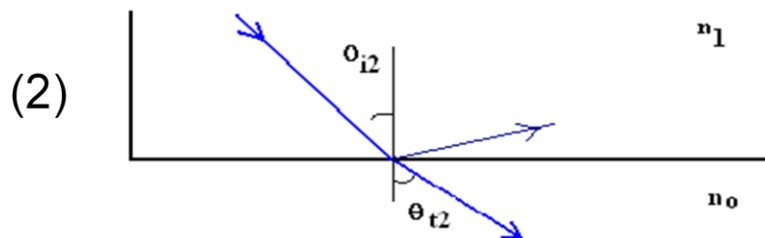
One clear example of refraction is the apparent depth of submerged objects. Let's take a moment to look at this.

Total Internal Reflection

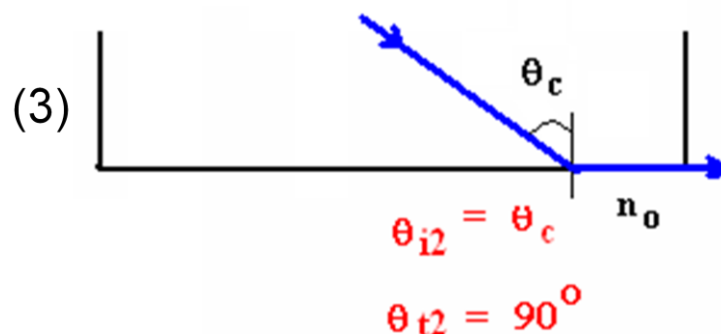
The reduction of apparent depth in water is one feature that tells us that light refracts away from the normal on emerging from a dense medium into air. Another is the possibility that the transmitted ray disappears entirely when θ_i exceeds a certain value, and we have total internal reflection.



In the second figure the angle θ_{t2} increases with θ_{i2} , and by the third figure, θ_{t2} has reached 90° at some value of θ_{i2} . The value of incidence angle at which the angle of refraction is 90° (where the emerging ray just grazes the boundary) is termed the **critical angle**, θ_c .



At values of incidence angle $> \theta_c$, the refracted angle $> 90^\circ$, i.e. the ray is totally reflected into the denser medium and no light is transmitted for $\theta_i > \theta_c$. We shall return to this phenomenon later.



The Critical angle of incidence θ_c is the θ_i value at which the transmitted ray grazes the interface, i.e. $\theta_t = \pi/2$.

For $\theta_i > \theta_c$, no transmitted ray – we have total internal reflection.

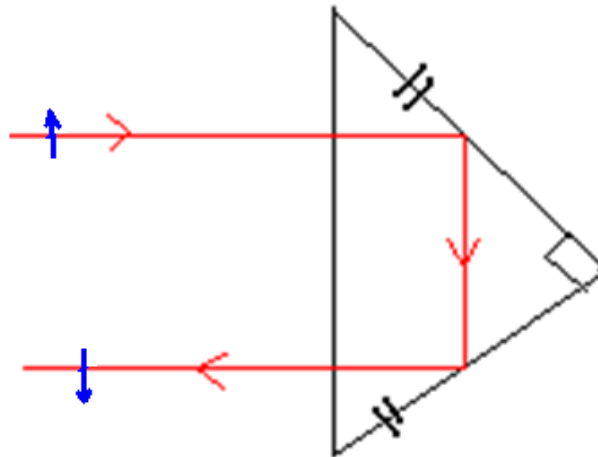
Snell's law at critical incidence reduces to $n_i \sin \theta_c = n_2$ (since $\sin \pi/2 = 1$)

Prisms

Some examples of prisms being used are:

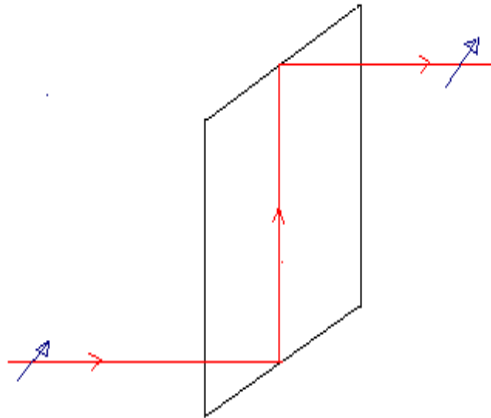
- For folding systems, reverting, inverting images e.g binoculars.
- For elevating beams

This section shows examples of rays diverted by total internal reflection. In the first figure we see how a right angle prism can be used to fold a beam by two sequences of total internal reflection.



Notice that the initial direction of the beam (first and second segment) is such that it is meant to go from glass to air. This makes total internal reflection possible as the glass is denser than air.

In the second figure, the beam is elevated with its direction unchanged. Together they illustrate how light can be bounced about inside material of higher refractive index.

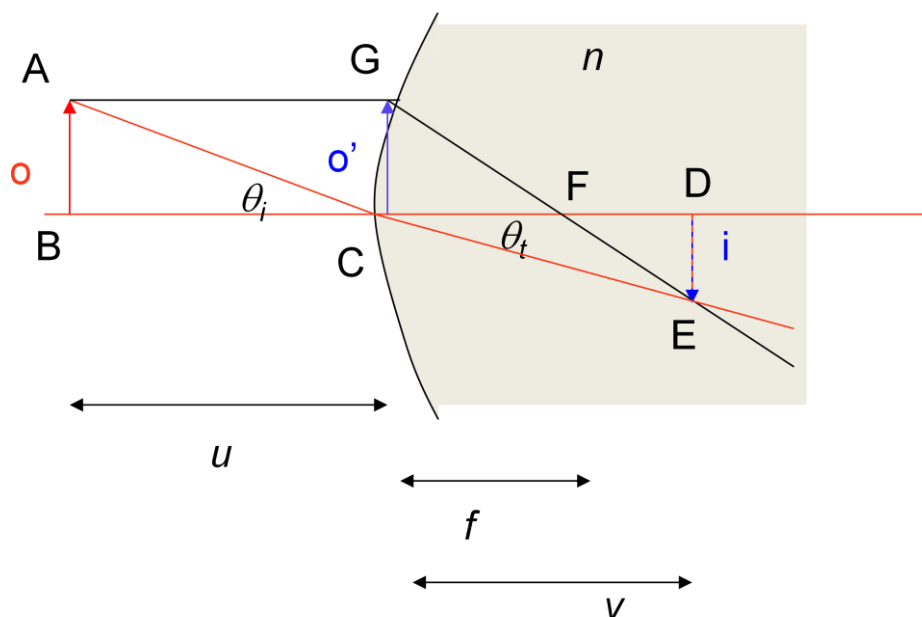


To address the problem of vision correction we now need to turn to refraction at curved surfaces.

By finding the critical angle for glass verify that the 45° - 90° - 45° glass prism illustrated will reflect light as shown.

Image Formation by Refraction

Just as reflection at a curved boundary gives rise to an image, so too does refraction, with the same caveats, that the curvature of the surface must be close to a paraboloid. The relation between object and image as a result of refraction is then similar to that for reflection except that the refracted and incident angle are not equal but obey Snell's law.



The magnification equation is therefore changed. For small angles we have $\theta_i = o/u$ from triangle ABC. Similarly $\theta_r = i/v$ from triangle CED. Snell's law relates the angles of incident and refraction in the small angle approximation as $\theta_i = n\theta_t$.

The focus equation is unchanged since it involves angles in a single medium; combining the two equations gives the lens makers formula but now with an extra factor of the refractive index.

From the Red triangles

$$\frac{o}{u} = \theta_i = n\theta_t = \frac{ni}{v} \quad \text{therefore} \quad \frac{i}{o} = \frac{v}{nu}$$

From the Blue triangles

$$\frac{i}{o'} = \frac{i}{o} = \frac{v-f}{f} \quad \text{therefore} \quad \frac{v-f}{f} = \frac{v}{nu} \quad \text{or} \quad \frac{1}{u} + \frac{n}{v} = \frac{n}{r}$$

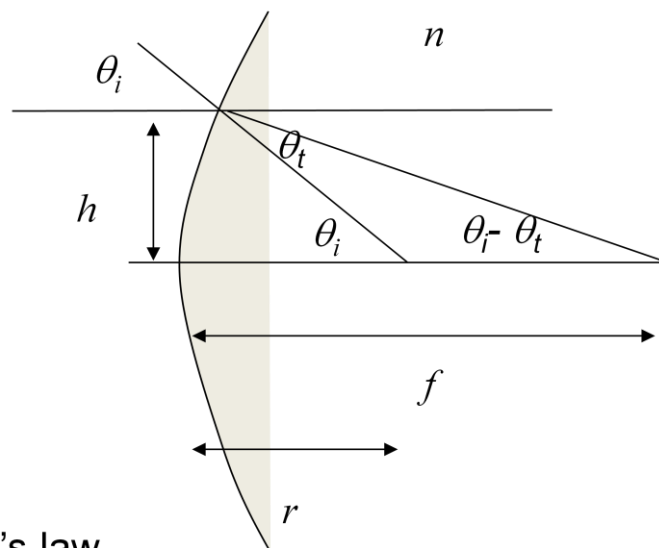
Relation between curvature and focal length

Finally in this section we see the calculation of the relation between the focal length and the radius of curvature for a medium of refractive index n embedded in a medium such as air or a vacuum with a refractive index of unity.

$$\theta_i = \frac{h}{r}$$

$$\theta_i - \theta_t = \frac{h}{f}$$

$$n\theta_t = \theta_i \quad \text{Snell's law}$$



The result is valid for rays close to the axis only, because we use Snell's law in the small angle approximation. Nevertheless the result is the key to image formation by optical systems: the focal length is independent of h – all parallel rays go through the focus. Just as

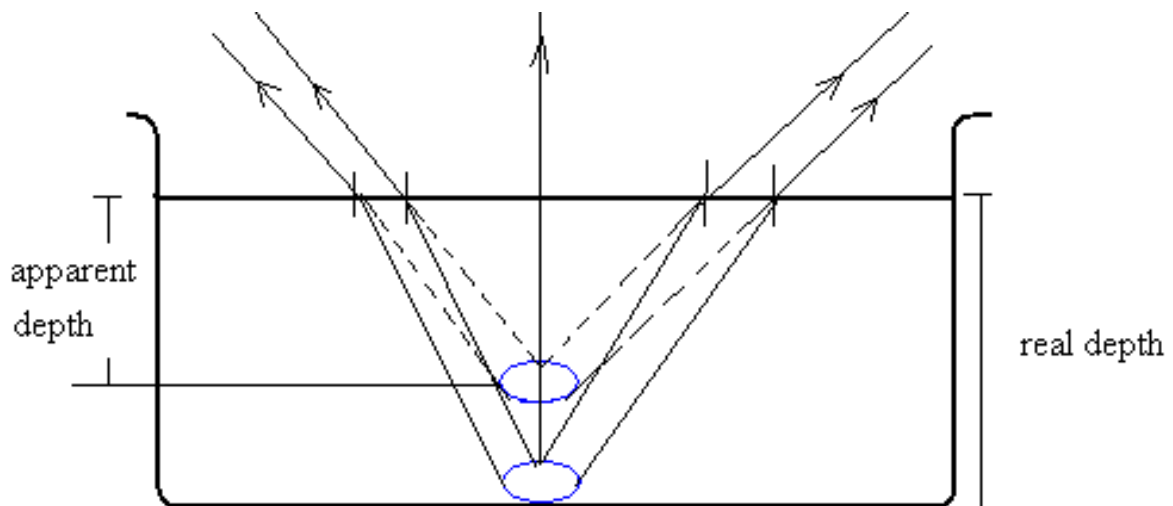
we saw in the case of reflection by curved mirrors, the existence of a focus is essential for the formation of an image.

Therefore: $f = \frac{nr}{n-1}$ and hence $\frac{1}{u} + \frac{n}{v} = \frac{(n-1)}{f}$

f is independent of h

Apparent Depth

This section explains the concept of apparent depth. It illustrates rays from an object at the bottom of a fish tank. As the rays emerge from the water/air interface they bend away from the normal.



An observer in air sees the rays as coming from an object located along the straight line. The refracted rays therefore make the object (and the basin) appear shallower than it is in reality. Since the surface is planar its radius of curvature is infinite so the object and image distances are related by:

$$\frac{n}{u} + \frac{1}{v} = 0$$

Summary

Snell's law:

$$n_i \sin \theta_i = n_t \sin \theta_t$$

Apparent Depth:

$$\frac{n}{u} + \frac{1}{v} = 0$$

Total Internal Reflection:

$$n_i \sin \theta_c = n_t$$

Image formation in a medium:

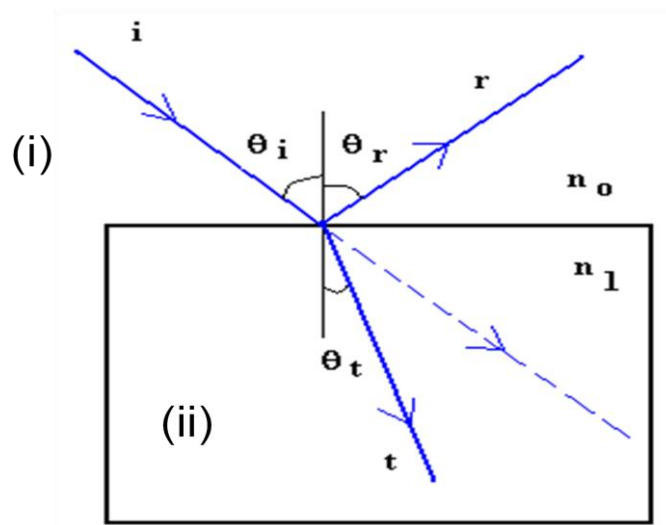
$$\frac{1}{u} + \frac{n}{v} = \frac{n}{r}$$

Relation of curvature and focal length

$$f = \frac{nr}{n-1}$$

SAQs.

1. What is the refractive index for water, air and for glass respectively
 - a) 1, 1.5, & 1.3
 - b) 1.3, 1 & 1.5
 - c) 1.3, 1, & 1.5
2. In the diagram below, $\theta_i = 60^\circ$ and $\theta_t = 35^\circ$. Label the media (i) and (ii) as:
 - a) air
 - b) water
 - c) glass



3. A fish rests at the bottom of a tank 1m deep while a fly hovers 2cm above the water surface. How far away does the fish think the fly is ?
 - a) 1m
 - b) 0.77m
 - c) 1.3 cm
4. What minimum angle of incidence on the glass/air interface is required for the ray to be totally internally reflected back into glass?
 - a) 42°
 - b) 50°
 - c) 90°

The answers appear on the following page

Answers

- (a) and (b) Incorrect: air molecules are polarisable but the density of air is sufficiently low that the polarisation per unit volume is effectively zero. So air has a refractive index of unity. The refractive indices of water and glass are found from tables, but you can remember the order from the fact that glass is denser.

(c) is correct – the refractive indices can be found in tables, but effectively follow the order of densities
- Correct: (i) (a) and (ii) (c): air and glass – the surrounding medium cannot have the largest refractive index because the incident ray bends towards the normal so the combination could be air + glass or air + water or water + glass. Snell's law gives $\sin \theta_t = 0.57$ for the air glass interface which gives $\theta_t = 35^\circ$. The water – glass combination gives 49° and the air – water combination 42° approximately.

Other answers: Snell's law is $n_i \sin \theta_i = n_t \sin \theta_t$. This gives $\sin \theta_t = 0.57$ for the air glass interface which gives $\theta_t = 35^\circ$. The water – glass combination gives 49° and the air – water combination 42° approximately, so (i) (a) and (ii) (c) is correct. Other combinations would give a refracted ray that bends away from the normal.
- a) Incorrect: the apparent depth is affected by refraction at the air water interface

b) Incorrect: You have used $d_{\text{water}} = d_{\text{air}}/n_{\text{water}} = 1/1.3 = 0.77$, but the refraction is from air to water, not the other way round

c) Correct: $d_{\text{water}} = n_{\text{water}} d_{\text{air}}$ – The light from the fly refracts towards the normal so the fly appears further away than it really is.
- a) correct - $\theta = \sin^{-1}(1/n) = \sin^{-1}(2/3)$

b) incorrect - $\theta = \sin^{-1}(1/n)$ - you appear to have used the refractive index for water

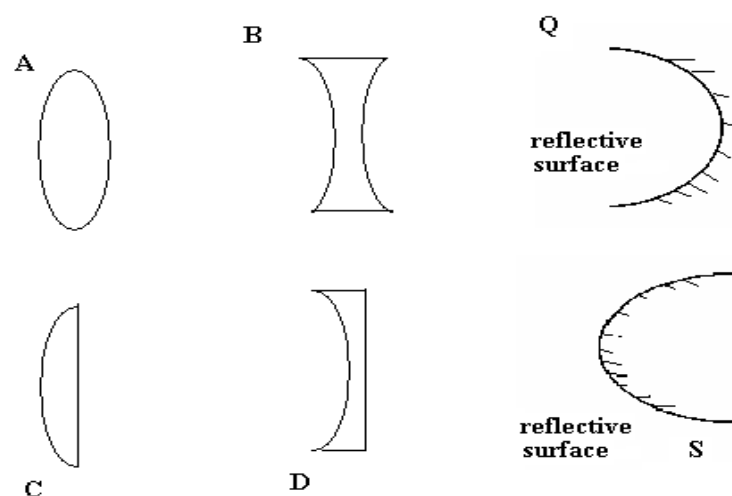
c) incorrect - 90° is the angle of refraction not the angle of incidence.

Lenses

In the light of our reading glass problem, and what we know that about refraction at interfaces, we'll now turn to lenses; that is we'll look at how light passes through a medium with two interfaces. We'll see several examples of non-planar optics, convex and concave surfaces and we'll deal with the rules of ray tracing and image formation.

Examples of Lenses

Lenses are refracting material of at least one curved interface. Other curved surfaces such as mirrors are also found in many applications e.g. driving mirrors. However, keeping focused to our problem of the reading glasses, we shall consider mainly lenses here. A lens of configuration A is termed bi-convex or positive lens whilst that of C is a biconcave negative lens. The positive and negative actually describes their focal lengths. C is a plano-convex and D a plano-concave lens.



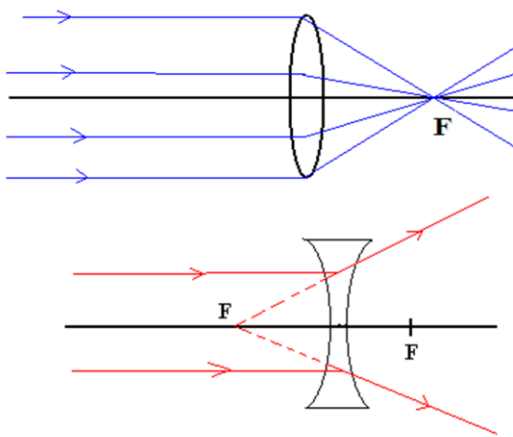
If one considers the curved surface of a lens as part of a sphere, then the radius of that sphere is radius of curvature, r , of the lens.

Lenses: Focal Length

In any configuration, lenses have a focal point defined by the way they refract light.

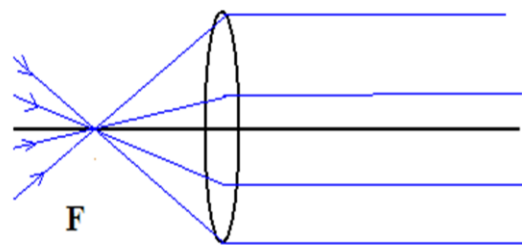
If rays parallel (and close) to the optical axis are incident on a biconvex lens they all converge to one point on emerging. For the biconvex case, rays pass through the lens diverging as though they originate from a common point F . On the other hand, if the rays were all passing through F before reaching the biconvex lens, they would emerge from the lens parallel to the optical axis. In the case of a biconcave lens the emerging rays from a parallel beam will appear to diverge from F .

Distant rays focused



Distant rays arriving parallel to the axis converge at (or appear to diverge from) focus F

Focus rays parallel

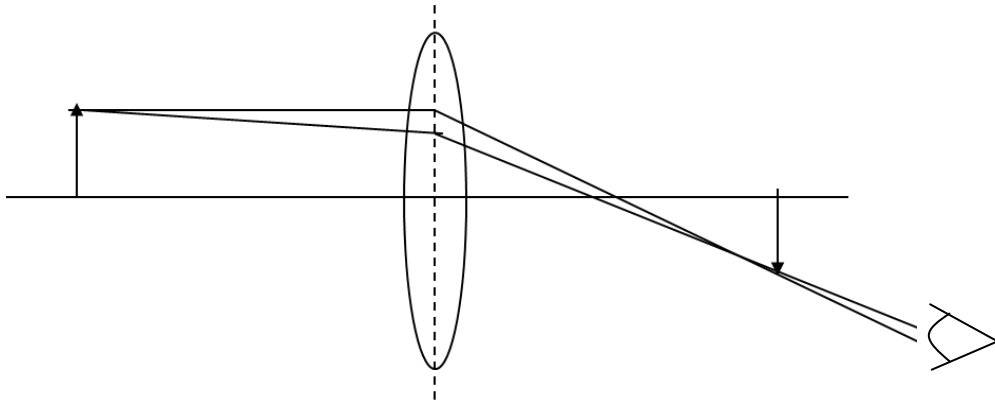


Rays passing through the focus F are parallel to axis on exit.

The point F is called the focal point of the lens and the distance from the focal point to the centre of the lens is termed the focal length, f .

Image formation by a lens

Just as for a reflecting surface, the existence of a focus for a lens guarantees that rays from a point on the object are brought together at a point to form the image. Thus rays diverge from the image point into the eye just as they would from the real object. The eye therefore sees the image.



Once we understand this there is no need to trace bundles of rays every time.

Draw a similar diagram for a diverging lens

Ray Tracing

As the figures illustrate, images are formed from objects by tracing the rays from all part of the object. Typically a few rays are sufficient to determine the image. All other rays, of course, are subject to same rule namely:

Ray tracing notes

- a. Rays parallel to the axis will always (appear to) pass through focus.
- b. Rays passing through the centre of lens never deviate
- c. The intersection of rays (a) and (b) determine image size and position.

Magnification $M = - \text{image size, } y_i / \text{object size, } y_o \text{ or } -s_i/s_o.$

The negative sign of the magnification shows that the image is inverted. The ratio of the respective distance also defines the magnification.

Image formation (+ve lens)

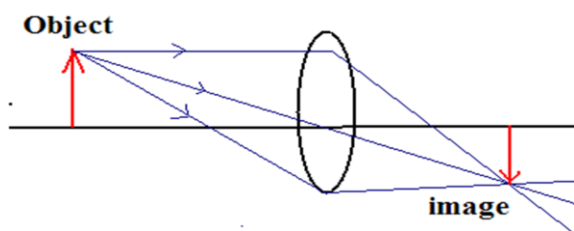
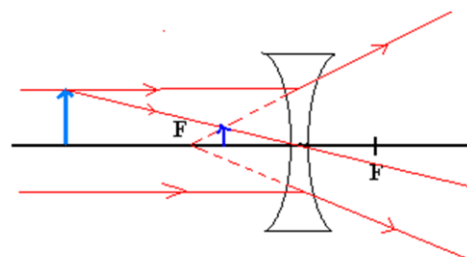
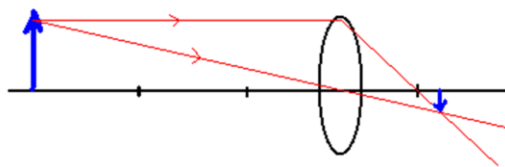


Image formation (-ve lens)

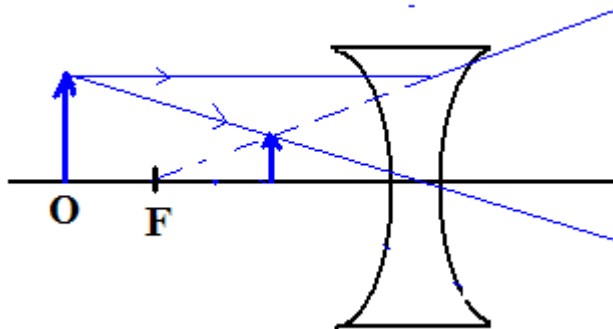
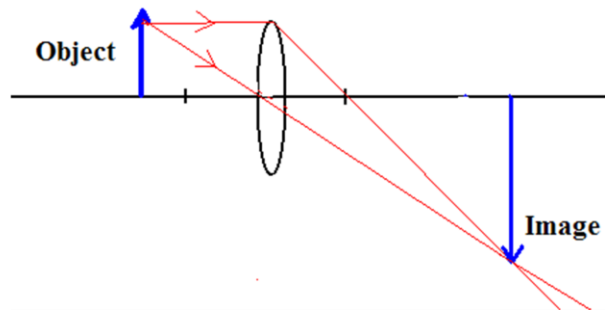


We observe that as the object is closer to the lens, the image recedes from it. The magnification is the ratio of the image size to the size of the object. Alternatively, the ratio of the image to object distances also defines the magnification. A negative sign signifies image inversion.

Distant object minified



Near object magnified

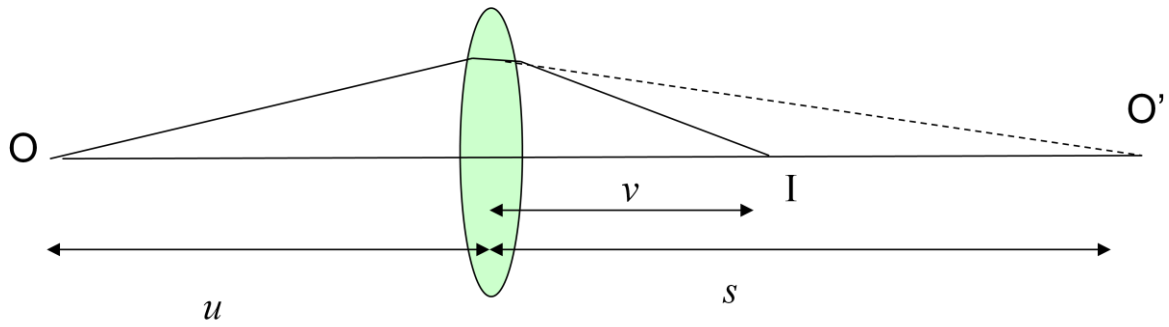


For a positive lens

- Image real
- Inverted
- Magnified, minified or same depending on object position.

Thin Lens Formula

To obtain the relation between image and object distances for a lens we consider the combined effect of two curved surfaces. We'll assume that all distances from the lens are measured from the same point – in other words the lens has zero thickness in this approximation. Thus our result will apply only to thin lenses, and of course to curvatures that do not depart significantly from a paraboloid.



At the first surface an intermediary object O' forms at a distance s from the lens. Eqn (1) applies, with r_1 the radius of curvature of the first surface. A final image forms from O' by refraction in the second surface for which eqn(2) applies. The object distance for the intermediary object O' is negative because it is on the same side as the image. Finally, we add (1) and (2) to get the overall result for the double surface.

$$\frac{1}{u} + \frac{n}{s} = \frac{n-1}{r_1} \dots\dots\dots(1)$$

$$\frac{n}{-s} + \frac{1}{v} = \frac{1-n}{r_2} \dots\dots\dots(2)$$

$$(1)+(2): \frac{1}{u} + \frac{1}{v} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \dots\dots\dots(3)$$

Lens Makers Formula

$$\left(\frac{1}{u} + \frac{1}{v}\right) = \left(\frac{1}{r_1} - \frac{1}{r_2}\right)(n-1) \quad (1)$$

We have just seen that for a refracting medium bounded by two curved surfaces, equation (1) holds for a ray travelling through the medium of refractive index n to air. In the case that the object is at a sufficient distance that the rays from it are effectively parallel the image forms at the focus. This gives us equation (2) which relates the focal length f , to the radii of curvature (r_1, r_2) of the lens.

$$\frac{1}{f} = \left(\frac{1}{r_1} - \frac{1}{r_2}\right)(n-1) \quad (2)$$

$$\text{If } u \rightarrow \infty, v \rightarrow f.$$

We can therefore finally write equation (3). This is the thin lens formula or lens makers' formula. It relates the image distance v to the object distance, u , and the focal length, f

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \dots\dots\dots(3)$$

Eye Correction

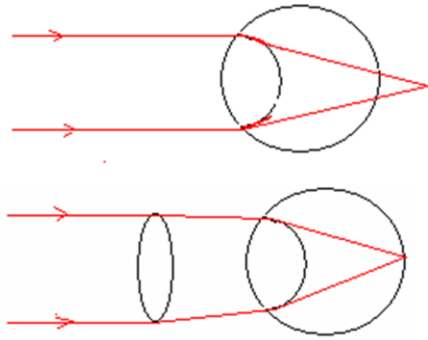
We can return now to the use of lenses to aid vision.

The first of the optical instrument we shall study is the vision correction glasses. It can be seen as a modern version of the Romans' discovery in the 13th century. We shall not go into medical detail in understanding what goes on in the eye and what defect is corrected by optics. It is enough to understand that the eye is composed of a lens which focuses light from objects to a region at the inner part of the back of the eye ball - - a region called the retina. A healthy eye can adjust its focal length (by making the eye lens thin or thick) so that objects far or near can be focused on to the retina.

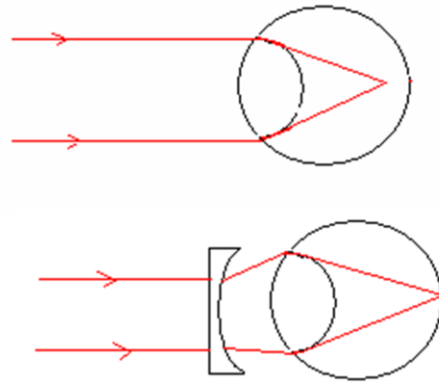
There is a closest distance of about 25 cm of the eye from which an object can normally be focused to the retina. This distance is termed the near point of vision. The near point varies slightly from person to person. For most people the far point of vision is at infinity.

Eyes are said to be "far sighted" if objects close to it are focused beyond the back of the retina. In other words, only distant objects can be visualized well and the near point is more than the normal 25 cm. As shown in the figure, converging lenses, with appropriately calculated focal lengths, when placed in front of the eye's lens, brings the rays to a focus on the retina.

On the other hand, with short sightedness, rays from distant objects are brought to a focus short of the retina. In order to correct this, slightly diverging lenses are placed in front of the eyes lenses, which extend the focus for distant rays to the retina, as shown in the figure.



Far sightedness. Distant object forms behind retina. Only near obj. seen. Positive lens decreases divergence. Compound focal length shortened.



Short sighted. Distant object forms short of retina. Only near obj. seen. Negative lens increases divergence. Compound focal length increased.

Determine your near point using a fixed character on a piece of paper. How close to your eye can you still see it properly? Can you estimate in cm? What about your far point of vision ?

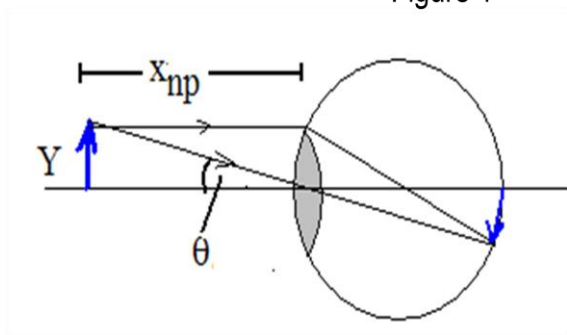
A Magnifier

This section further illustrates how a simple magnifying glass aids vision. The angular magnification is the ratio of the angle subtended at the eye by the object when aided relative to the angle when unaided because this gives the ratio of the sizes of the unaided and aided images on the retina.

In fig.1, we see that the angle subtended by the unaided eye by an object of height Y , at the near point, X_{np} , is Y/X_{np} . When a lens of focal length f , assumed to be in contact with the eye, is used as vision aid, the angle subtended by an object at the focus of the lens is Y/f . This object appears to the eye at infinity

Unaided eye:

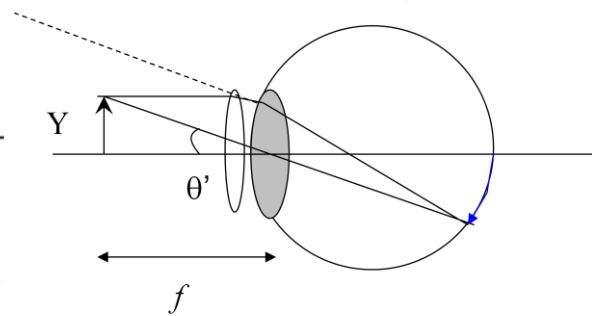
Figure 1



$$\tan \theta \approx \theta = \frac{Y}{X_{np}}$$

Aided eye

Figure 2



$$\tan \theta' \approx \theta' = \frac{Y}{f}$$

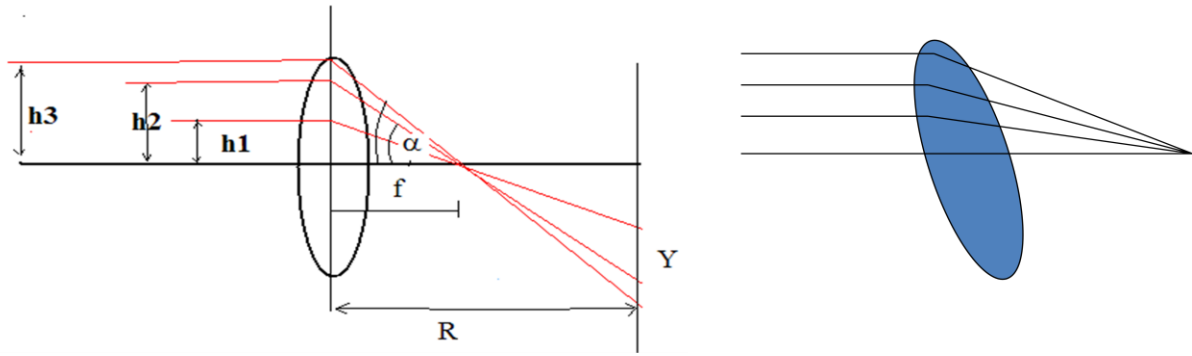
The magnifying power of a lens is the ratio of the angle subtended by the object at the near point with and without the magnifier. This is because the ratio of these angles determines the ratio of sizes of the image on the retina. As shown, the magnifying power of the magnifying lens is:

$$M = \theta' / \theta = \frac{Y}{f} \cdot \frac{X_{np}}{Y} = \frac{X_{np}}{f}$$

The magnification arises because with a suitable lens present, the image can be brought closer to the eye: clearly, the lens must have a focal length less than X_{np} or 25 cm.

Focal Length and Angled Rays

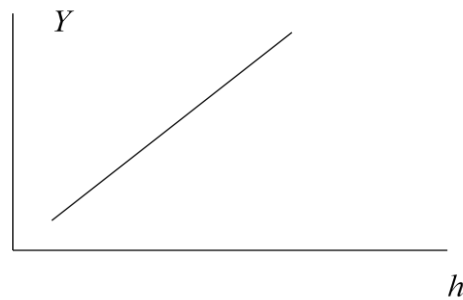
We can now return to one of our original problems: the way in which modern spectacles work. All of our drawings and the drawings in text books show the object, lens and image nicely aligned. However, when we wear spectacles we assume they will be used at all angles. The key to their use is that the lens shape is such that it doesn't matter what angle the rays strike the lens: we shall always get an image.



The figure shows an easier way of looking at this: for a thin parabolic lens the rays parallel to the axis all pass through the focus. The image on the screen is therefore a linearly scaled version of the object, no matter where the object is viewed through the lens. Because we are dealing with a thin lens, tipping the lens up simply means that the rays strike the lens at different angles to the normal, but the relationship between them is unchanged: the spectacle lenses still produce a faithful image.

$$\tan \alpha \approx \alpha = \frac{h}{f} = \frac{Y}{R}$$

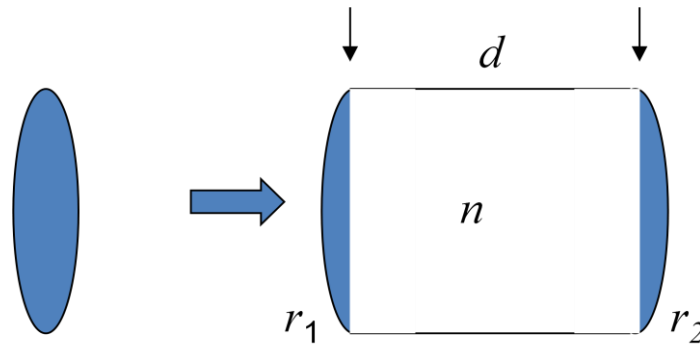
So $Y \propto h$



In fact, spectacle lenses are not ground as paraboloids, but as much simpler spheroids, so do not work perfectly in this way – nevertheless spectacle lenses are small enough that the difference is not noticeable.

Thick Lenses

Our original problem involved not thin modern spectacles but the use of spherical lenses as reading glasses. When the thickness of a lens between the curved surfaces, d , is significant, it no longer behaves as a thin lens. The theory of thick lenses is quite complicated, so we'll just pick out some results that will address this problem. Rather than consider a spherical lens, we'll start with a lens that is thick because the two refracting surfaces are separated by a block of glass.



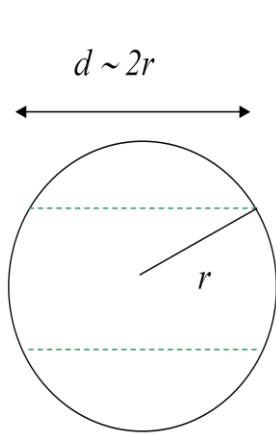
The figure shows surfaces with radii of curvature r_1 and r_2 separated by a block of the same material of thickness d and refractive index n . There are some complications now in defining where the object and image distances, u and v , and focal lengths are measured from, but if the curved surfaces are not too thick we can assume distances are measured from the plane surfaces of the block as indicated. The formulae then give the focal length f of the configuration and the magnification M of an object.

$$\frac{1}{f} = (n-1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \rightarrow \frac{1}{f} = (n-1) \left[\left(\frac{1}{r_1} - \frac{1}{r_2} \right) + \frac{(n-1)d}{nr_1r_2} \right]$$

$$M = -\frac{f}{u-f}$$

Focal Length of a Spherical Lens

We can apply our thick lens formula to the spherical reading glasses if we confine our attention to the image formed in the central portion of the lens. As the figure shows, the sphere is then approximated by two end regions of radius of curvature r and $-r$ and a region separating them of length d approximately equal to $2r$. The focal length of a glass sphere of radius r is therefore about $(3/2)r$.



$$\frac{1}{f} = (n-1) \left[\left(\frac{1}{r_1} - \frac{1}{r_2} \right) + \frac{(n-1)d}{nr_1r_2} \right]$$

$$\frac{1}{f} = \frac{2(n-1)}{r} - \frac{2(n-1)^2}{nr}$$

$$= \frac{2(n-1)}{nr}$$

$$= \frac{2}{3r} \quad (1) \quad \text{for glass}$$

With the object close to the lens, the image is at a large distance so $v \sim \infty$ and $u \sim f$. The magnification then reduces to v/u . The magnifying power is given by taking the image to be at the nearpoint $v = X_{np}$. The ratio v/u is then equal to the magnifying power X_{np}/f . Taking f from equation (1) and replacing r in cm by the diameter of the lens in mm gives us a magnifying power of $333/d$.

Magnification:

$$M = -\frac{f}{u-f} = -\frac{v-f}{f}$$

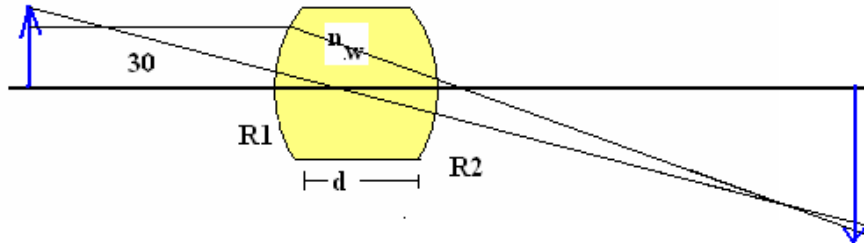
But $v \gg f$ so $u=f$ and $M = v/u$

Magnifying power: X_{np}/f

$$= \frac{25 \times 2}{3r} = \frac{1000}{3d(\text{mm})} = \frac{333}{d}$$

Summary

- Lenses are refracting material of at least one curved interface.



- The glass globe is a form of lens
- Lens maker's formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

- Thin lens focal length:

$$\frac{1}{f} = \left(\frac{1}{r_1} - \frac{1}{r_2} \right) (n_2 - 1)$$

- Thick lens focal length:

$$\frac{1}{f} = (n-1) \left[\left(\frac{1}{R_1} - \frac{1}{R_2} \right) - \frac{(n-1)d}{nR_1R_2} \right]$$

SAQs

1. Why does the glass object shown not behave as either a converging or diverging lens?
 - a) Because it doesn't taper at the top and bottom
 - b) Because the bending of rays at the first surface is exactly cancelled by the second.
 - c) Because it is too thin

2. At what object distance from the lens are object and image same size?
 - a) Same distance as focal length.
 - b) twice focal length.
 - c) half focal length

- a) What is the magnification of a spherical water lens of diameter d mm?
 - a) $333/d$
 - b) $250/d$
 - c) $25/d$
 - d) $125/d$

The answers appear on the following page

Answers

1. a) Incorrect – tapering the edges would not affect the central aperture
b) Correct – the important point about a thin lens is that its surfaces are curved differently, usually such that the radii of curvature have opposite signs.
c) It's true that in order to accommodate different radii of curvature for the two faces it is necessary to make the centre thicker (biconvex lens) or thinner (bi-concave lens). But this is not the reason why these objects behave as lenses.

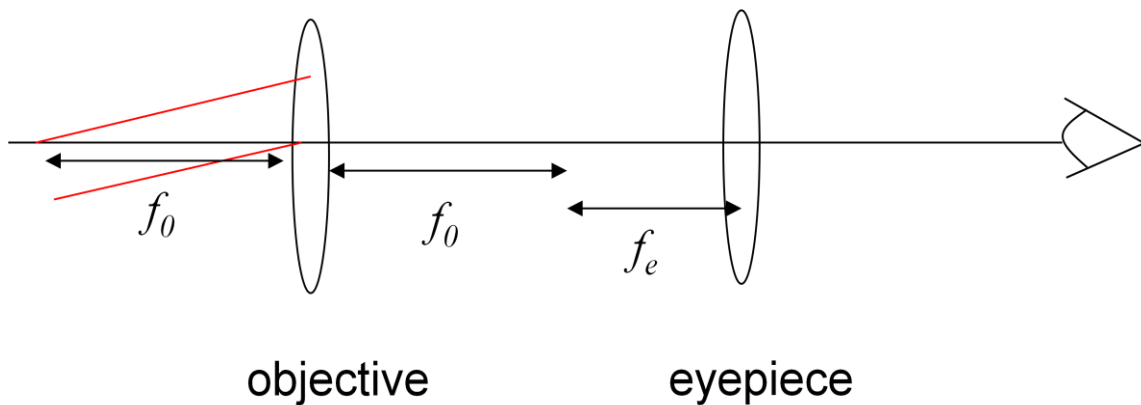
2. a) Incorrect – an object at the focus gives an image at infinity and vice-versa
b) correct – to be the same size we must have $u=v$. The lens formula then gives $2/u = 1/f$ so $u = 2f$.
c) Incorrect – an object closer than the focus does not give a real image.

3. a) Incorrect – this is the value for glass: the magnification depends on n , the refractive index, which is different for water.
b) Correct – this is the value for a refractive index of $4/3$
c) Incorrect – you appear to have forgotten to convert d to mm
d) Incorrect – you appear to have used d as the radius not the diameter

Additional Problems

Problem 1: Telescopes

In a telescope the lenses are arranged such that they are separated by the sum of their focal lengths



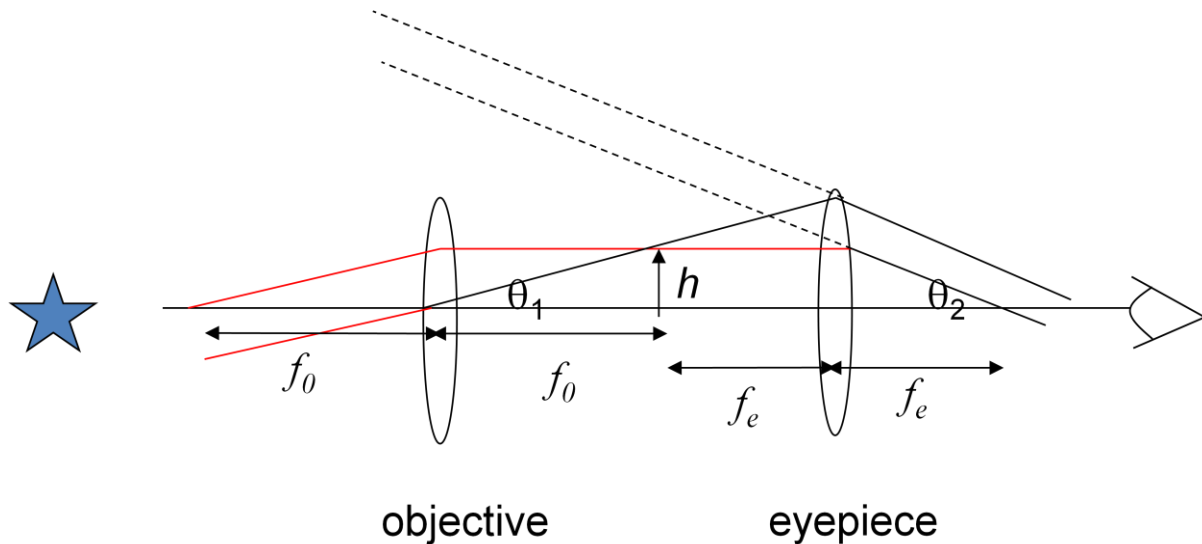
The figure shows a ray from the centre of the object and rays from one edge: these are parallel because the object is at infinity.

Show that the final image is seen at infinity, and that the magnification is f_0/f_e

As a hint, recall that the magnification is the ratio of the angular size of the object seen with the telescope to the angular size without the telescope.

Answer:

The back focal point of the objective coincides with that of the eyepiece. This is the basic arrangement of lenses in a telescope. In a telescope the object viewed is often far away effectively at infinity.

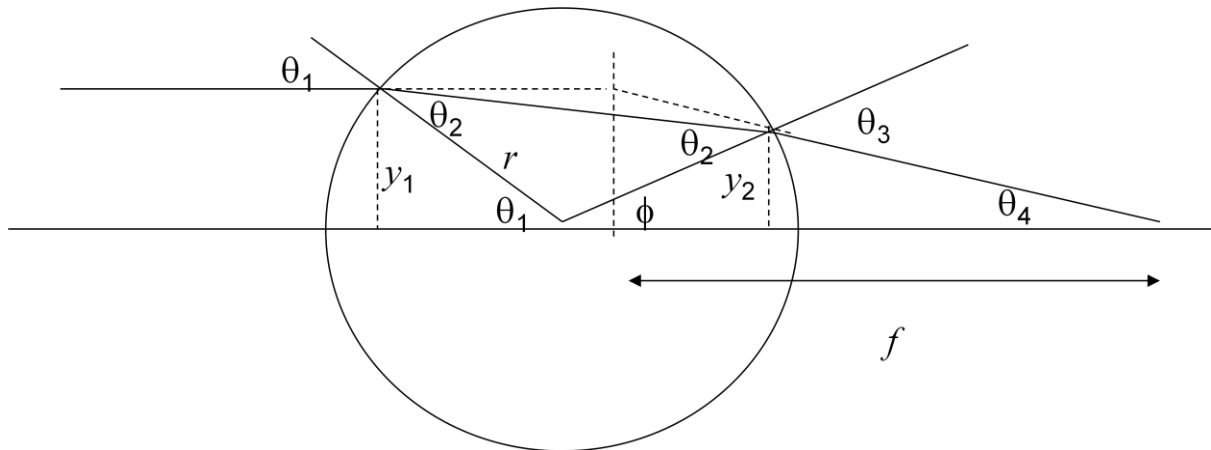


The figure shows the central ray and rays from one edge – these are parallel because the object is at infinity. The magnification is the ratio of the angular size of the object as seen with the telescope to the angular size without the telescope.

$$\begin{aligned} \text{The magnification} &= \frac{\text{Angular size of the image through the telescope}}{\text{Angular size of the image without the telescope}} \\ &= \theta_2 / \theta_1 \\ &= \frac{h / f_e}{h / f_0} = \frac{f_0}{f_e} \end{aligned}$$

Problem 2: Spherical Lenses

This question is here is to justify our approximate formula for the focal length of a spherical lens. This is done by tracing the angles and using Snell's law in the small angle approximation $n_1\theta_1 = n_2\theta_2$. Begin by noting that $1/f = \theta_4/y_1$.



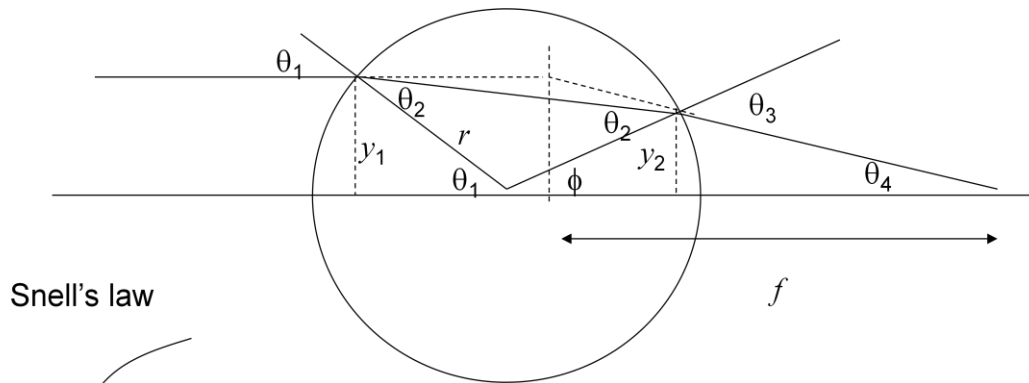
f is measured from the intersection of the incoming rays parallel to the axis and corresponding outgoing rays. It can be shown that this is actually the centre of the sphere but we shall not do so here.

The problem is to show that for a spherical lens of radius r and refractive index n

$$\frac{1}{f} = \frac{2(n-1)}{nr}$$

Answer:

Snell's law at the first refraction gives $n\theta_2 = \theta_1$ if we approximate $\sin\theta$ as θ for small angles. θ_4 and ϕ are the internal angles of the triangle with θ_3 as external angle. $\theta_3 = \theta_1$ by Snell's law again – or by symmetry. $\phi + \theta_1$ is the external angle of the equilateral triangle with θ_2 as the two internal angles. Putting this together gives the required result:



Snell's law

Angle chasing: $\theta_4 = \theta_3 - \phi$

By Snell's law $\theta_3 = \theta_1$

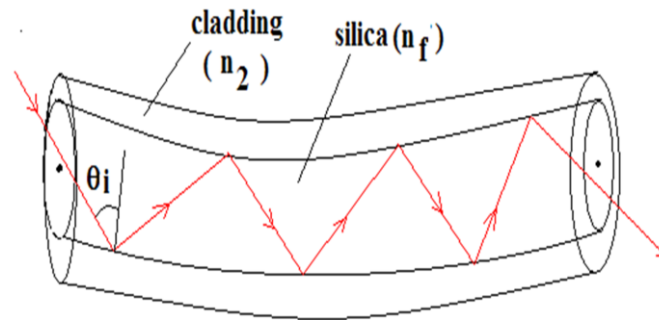
$$\phi = 2\theta_2 - \theta_1$$

So: $\theta_4 = \theta_1 - (2\theta_2 - \theta_1) = 2(\theta_1 - \theta_2)$

$$\frac{1}{f} = \frac{\theta_4}{y_1} = \frac{2(\theta_1 - \theta_2)}{r\theta_1} = \frac{2}{r} \left(1 - \frac{\theta_2}{\theta_1} \right) = \frac{2}{r} \left(1 - \frac{1}{n} \right) = \frac{2}{r} \left(\frac{n-1}{n} \right)$$

Problem 3: Optical Fibres

The figure shows a section of a rod, say of glass of refractive index n_f , clad in material of low refractive index, n_c . This is the structure of a modern optical fibre.

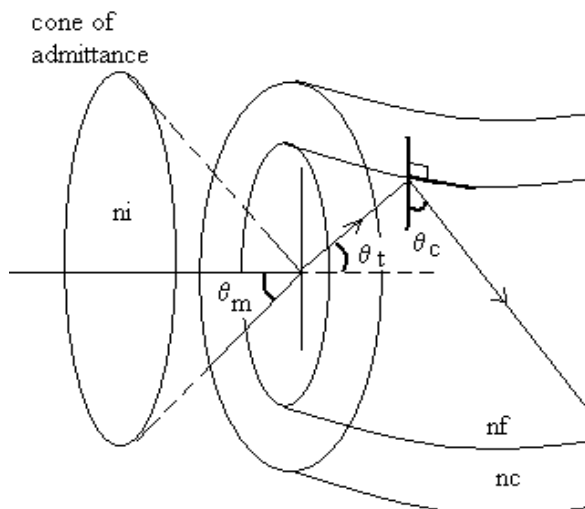


$$\theta_i > \theta_c \quad n_c < n_f$$

When light is incident at one end of the fibre, it may hit the glass/cladding interface such that it should transmit into the cladding from the glass. However, because the cladding has a lower index of refraction than glass, the light is totally internally reflected back into the glass, also at an angle such that it bounces to the opposite glass/cladding interface where it is again reflected internally. In this way light bounces internally from one end of the fibre to the next.

There is a wide range of angles (forming a shallow cone of admittance) within which light incident on the the end of a fibre is transmitted to the other end. The angle of the half cone, θ_m , within which light can be admitted is determined by the refractive index of the cladding n_c and the glass n_f . Since θ_m corresponds to the critical angle at the silica /cladding interface, the problem can be solved by chasing back the critical angles, remembering that $\cos\theta$ is the

square root of $1 - \sin^2\theta$.



Show that the angle of the cone of admittance is given by:

$$\sin\theta_m = \frac{1}{n_i} \sqrt{n_f^2 - n_c^2}$$

Overall summary

- Images are formed by tracing rays back to the eye
- The angle of reflection equals the angle of incidence
- For curved reflecting surfaces that approximate a paraboloid the lens formula applies
- Snell's law governs the relation between the angle of refraction and the angle of incidence
- The thin lens formula is derived from Snell's law for rays close to the axis
- The thin lens formula is modified for spherical lenses
- The magnification by an optical device is the ratio of the angular size of an object with and without the optical aid

Meta tags

Author: Samuel Atarah.

Owner: University of Leicester

Title: Enhancing Physics Knowledge for Teaching – Geometrical Optics

Keywords: Reflection; Refraction; Lenses; Optics; Light; sfoer; ukoer

Description: This section is concerned with Optics; we shall be interested in the behaviour of visible light as it interacts with plane and curved surfaces.

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Language: English

Version: 1.0

Additional Information

This pack is the Version 1.0 release of the module. Additional information can be obtained by contacting the Centre for Interdisciplinary Science at the University of Leicester.

<http://www.le.ac.uk/iscience>

