

## Part 2 Fields and Waves

### Session 4

### Current, Electricity

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# Welcome

Welcome to session 4 of the Physics programme. In this session we will be introducing electric currents. We will consider how an electric current behaves in DC and AC circuits, and how these circuits are described with circuit components like resistors, capacitors and inductors.

In this session we shall be using two problems to cover the learning outcomes, one for DC current and one for AC circuits.

## Session Author

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Session Editor – Tim Puchtler

# Learning Objectives

- Define DC circuits and the way in which current flow in them
- Use circuit diagrams and symbols correctly
- Understand basic electrical components (resistors, inductors, and capacitors)
- Define AC circuits and the way in which current flows in them
- Draw analogies with mechanical response phenomena

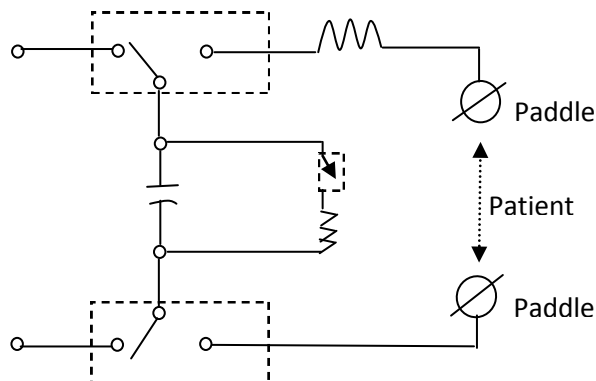
# The Problem

Heart defibrillators, which are used to restore a regular heart beat, stimulate the heart to contract by delivering a short current pulse of duration 20 ms. In one type of defibrillator a capacitor is charged to a suitable voltage and then discharged through the patient's chest with the aid of two large electrodes. The defibrillator needs to be able to deliver pulses of up to 360 J to patients with transthoracic resistances ranging up to 150 ohms.

Estimate values for capacitance and voltage needed to cope with these requirements.



Image <sup>1</sup>



<sup>1</sup> AED Training and CPR Training, Frederick Md Publicity as posted on [www.Flickr.com](http://www.Flickr.com), Creative Commons Licensed

The figure shows a typical defibrillator circuit. We'll see how to analyse this using a simpler version.

Our learning issues will involve understanding resistance and capacitance, and we'll also need to make a connection with the pulse duration and with energy. We'll start with resistance. But we won't immediately begin with Ohm's law,  $V=IR$ , because there's a lot to be said in order to avoid confusion with this law; surprisingly it can be particularly troublesome in the classroom.

# Resistance

## Circuits - Introduction

The picture shows a huge number of Christmas tree lights.



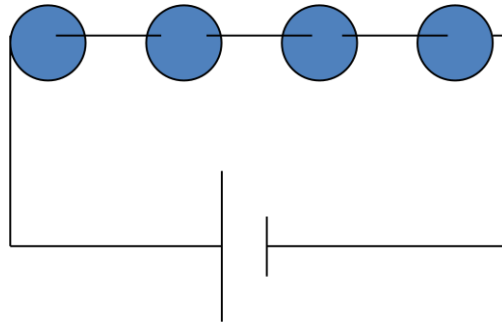
Image:<sup>2</sup>

With this number it is quite likely that one or more of the lights will fail at any one time. But the others stay on. How is that?

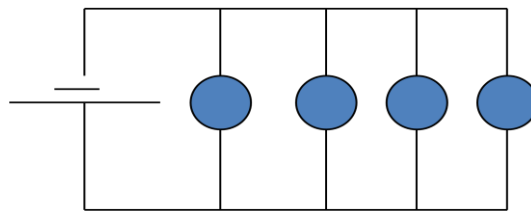
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<sup>2</sup> The National Christmas Tree, 12/14/1978, The U.S. National Archives' Photostream, as posted on [www.Flickr.com](http://www.Flickr.com), Creative Commons Licenced.

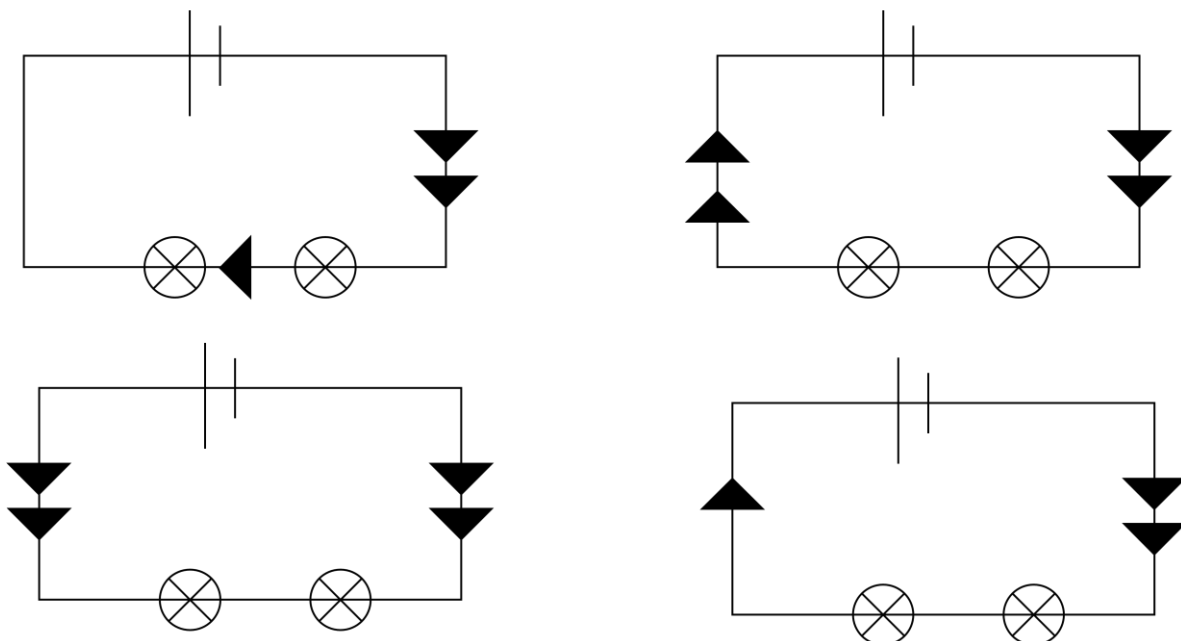




Which bulb is the brightest?

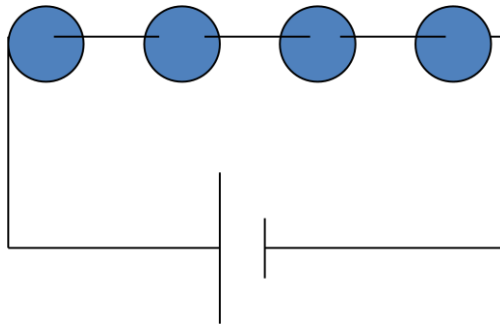


In order to understand what is really meant by 'potential difference (voltage)' or current, it is useful to understand the flow of electrons in the wires. It is often said that water flow along a river is analogous to current flow in a circuit: the potential difference is as the drop in height between two points, the resistance represents anything slowing the flow of water, and the current is, well, the current!

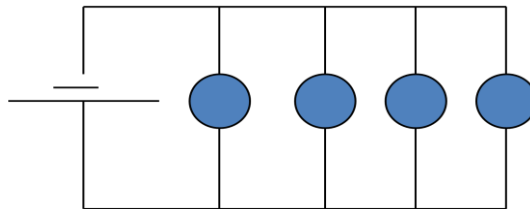


Consider the diagrams above. Which best describes the analogy of the water flow? Can you think of any reason the water flow analogy may be misleading?

## Some Common Misconceptions



Electricity is used up, so the last bulb is the dimmest



After the current has flowed through the first bulb there is less left to flow through the second so the lamps are progressively dimmer

Let's begin with a couple of common misconceptions. In the first picture the misconception is that electricity is used up, so the last bulb is the dimmest. The analogy is along the lines that current is forced through each resistance like water is forced through constrictions in a pipe until the pressure is diminished – so the last bulb is dimmer. The point is that this is not “obviously” wrong – the world could have been like this, but it isn't. So our learning issue is to find out enough about how electricity works to understand why.

In the second illustration the claim is that after the current has flowed through the first bulb there is less left to flow through the second so the lamps are progressively dimmer.

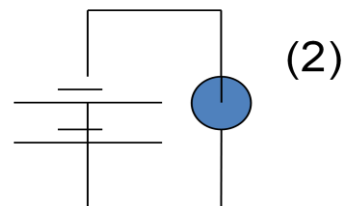
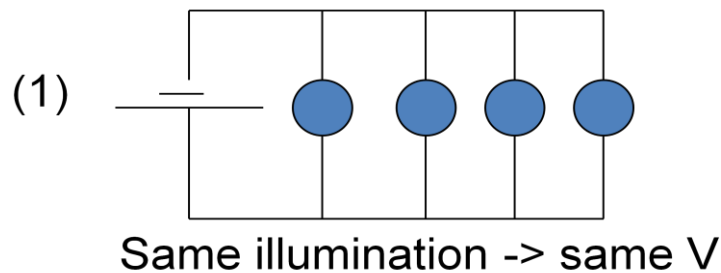
We understand that actually the amount of current peeling off at each opportunity is the same, but can we explain why? One way might be to understand that the current doesn't just come out of the battery – it's there in the circuit to start with; the battery just drives it round.

## A phenomenological approach to Ohm's Law

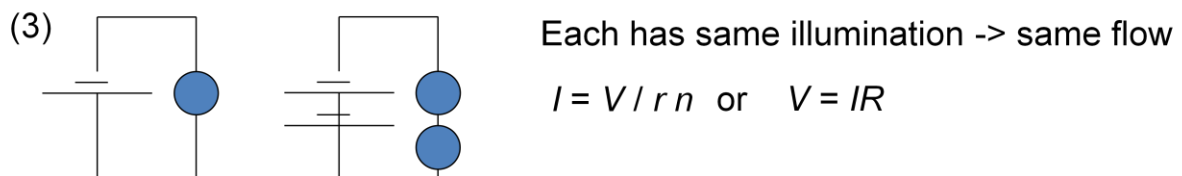
We should be able to establish an understanding of current electricity without a microscopic viewpoint by some carefully chosen thought experiments (or real experiments if you prefer). We call this a phenomenological approach to current electricity. Later, once we understand the structure of matter, we might try to find a microscopic explanation.

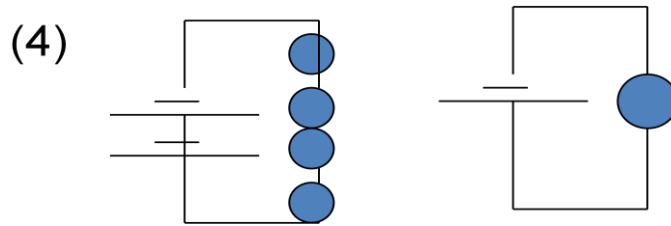
Our first observation is that there must be a complete circuit for the bulbs to light. This suggests that in an electric circuit something is flowing. This is a reasonable hypothesis, not a proof, which comes only when fundamental nature of electricity is understood. We call the flow a current – it's just a name at this stage. If there is a flow it is reasonable that something is driving it. Call this potential difference.

In illustration (1) the bulbs are equally illuminated. Therefore the potential difference is the same across each and there is a quantity  $V$  that is constant from one link to the next. Potential Difference is therefore a property of the battery. The battery drives a flow (or current) through the bulbs.



Bigger pd drives a bigger current



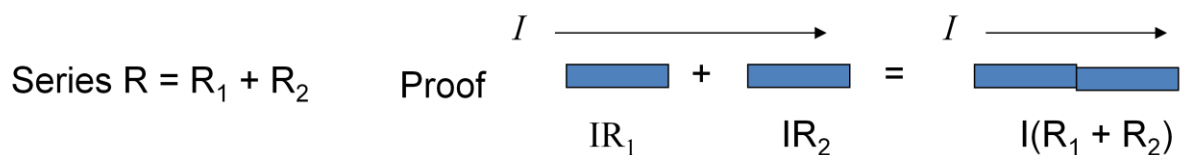


Hence Flow,  $I = V/R$  and dissipation proportional to  $I^2R = V^2/R = VI$

## Summary of formulae for resistors:

This section summarizes properties of resistors in series and in parallel.

In series the voltages across resistors add and the currents are the same through each. Hence the resistances sum.



In parallel the currents add and the voltages are the same across each. Hence the inverse resistances (or conductivities) add and the overall resistance is the harmonic mean.



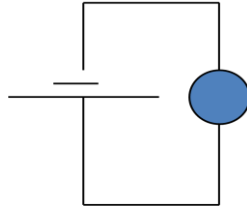
The SI units are volts, amps and ohms for voltage or potential difference, current and resistance respectively.

In flowing for a time  $t$  a current  $I$  transports a quantity (“charge”)  $q = It$  from one pole of the battery to the other. This is not “used up”.

**Current is not used up, and yet batteries do not last forever. What *is* “used up”?**

## What is used up?

Consider a simple circuit of a battery and a lamp.



The lamp emits energy. Therefore work must be done in transporting the charge round the circuit. This work comes from the battery. The battery stores energy in the form of available work. This is what is 'used up'.

**What type of energy does the battery store?**

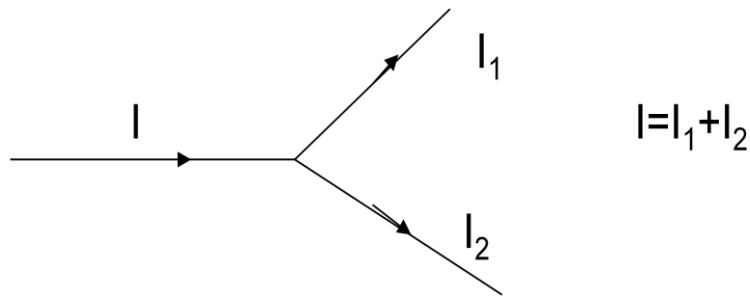
**Is it chemical, electrical, potential, kinetic?**

## Summary

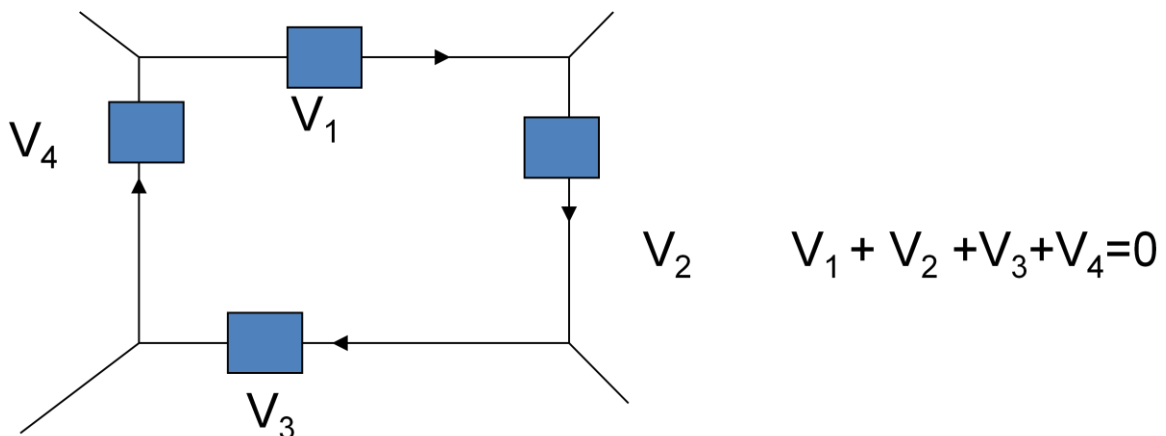
When a DC circuit consists of a network of elements in parallel and in series, it is no straighter forward to determine the equivalent resistance or current especially if there is more than one battery. In such cases the properties of DC circuits can be summarized in what are called Kirchoff's laws. The first of the two laws states that the current is conserved at a junction, that is, the sum of current entering a junction = sum of current leaving it. The second law, the loop law is about conservation of electric potential. It states that the sum of potential gained or lost by a charge in going round a closed loop is zero.

Kirchoff's Laws:

1. Current is conserved at a vertex

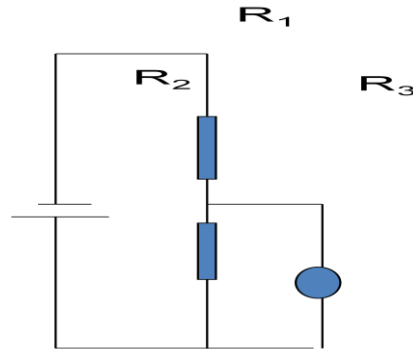


2. Sum (IR) = 0

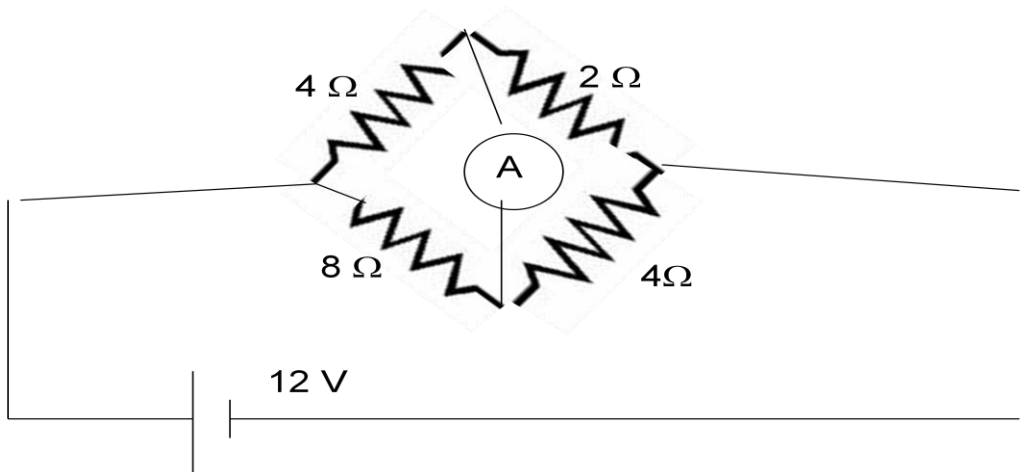


## SAQs

- Potential divider: The battery has an emf of 2V. What does the voltmeter across the second resistor measure when the resistors are  $R_1 = 2\text{k}\Omega$  and  $R_2 = 4\text{k}\Omega$ ? Enter your answer without units as an integer or a fraction in volts.



- If the voltmeter has a finite resistance  $10\text{ k}\Omega$  what is the total resistance of the circuit? Enter your answer in  $\text{k}\Omega$  as a fraction without units.
- Which of the following is a reasonable estimate of the resistance of a  $60\text{ W}$  domestic light bulb:  
(a)  $10\text{ k}\Omega$  (b)  $100\text{ k}\Omega$  (c)  $4\ \Omega$
- To save money it is suggested I cut the  $1\text{ kW}$  element of my electric heater in half. Am I right to reject the suggestion? (a) Yes (b) No
- What does the ammeter read in the following diagram?



- 0
- 1A
- 3A

The answers appear on the following page

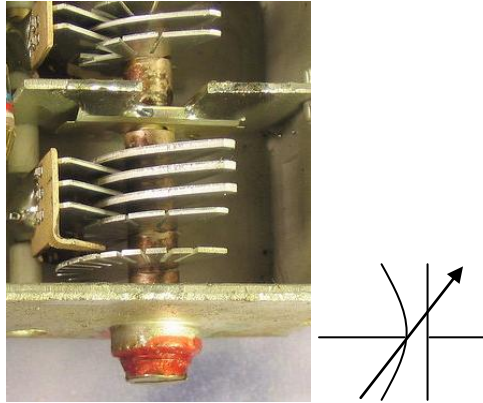
## Answers

1. Correct answer is  $4/3$ . The voltmeter can be taken to have infinite resistance since we are not told what it is. The potential drop is in the ratio of the resistances so  $4/6$  or  $2/3$  of it is across  $R_2$ .
2. Correct answer is  $34/7$  k $\Omega$ . The two resistors  $R_2$  and  $R_3$  are in parallel so give a total of  $(1/10 + 1/4)^{-1} = 40/14 = 20/7$  to which must be added  $2$  k $\Omega$  or  $34/7$ .
3. (a) Wrong: this assumes 100% efficiency which is not reasonable.  
(b) Correct – this is probably the nearest assuming 10% efficiency  
(c) Wrong – the formula is  $V^2/R$
4. (a) Yes is correct: the formula for power is  $V^2/R$ , so halving the resistance doubles the power by taking a bigger current.  
(b) Wrong: half an element has lower resistance, hence a bigger current. Using any of the forms of Joule's law for the power will show that the power goes up if the resistance goes down.
5. (a) Correct: the PD across the top branch and bottom branch must be the same. Across the top  $4/6$  ths of the drop occurs across the  $4 \Omega$  resistor. Across the bottom  $8/12$  ths of the drop occurs across the  $8 \Omega$  resistor. Since  $4/6 = 8/12$  the drop in each case is the same. Hence the pd across the ammeter is zero, so there is no current.  
(b) Wrong: you may have got this by reasoning that it looks as if  $2$  A goes through the upper route and  $1$  A through the lower leaving  $1$  A to go through the ammeter. In fact the PD across the left part of both branches is the same ( $4/6$  of the total for the upper branch and  $8/12$  for the lower), so there is no current flow.  
(c) Wrong: you may have argued that the total resistance of the circuit is  $4 \Omega$  (adding series and parallel branches) so  $3$ A flows, which is what an ammeter must measure.



# Capacitors

In the first section we looked at the properties of resistors. The second element in our defibrillator problem is the capacitor. Let's turn to that now.



Image<sup>3</sup>

A Capacitor stores energy in form of positive charge on one plate and negative charge on the other. The amount of charge on either plate for a potential difference of 1 volt is the capacitance of the capacitor. If  $V$  is in volts and  $q$  in Coulombs the  $C$  is in Farads.

$$\text{Capacitance } C = q/V$$

**The unit of Capacitance is the Farad**

The energy stored in a capacitor is  $\frac{1}{2} q^2/C$ . This is the work done in separating a charge  $q$  across a potential  $q/C$ . Since the potential varies as the charge builds up, we have to integrate to get the work done, which gives us the factor of  $\frac{1}{2}$ . Alternatively, the work done can be found by imagining the current flowing through the circuit as the charge builds up on the capacitor plates. The work done is  $VIdt$  integrated over time, or  $Vdq$  integrated over the charge.

$$\text{Energy} = \int VI dt = \int qdq/C = \frac{1}{2} q^2/C$$

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<sup>3</sup> Variable Capacitor, Robbie1's Photostream, as posted on [www.flickr.com](http://www.flickr.com/photos/robbie1/4644309/), Creative Commons Licenced. <http://www.flickr.com/photos/robbie1/4644309/>

What is the voltage drop across an uncharged capacitor? How much effective resistance does it offer to direct current flow?

How much current is flowing across a fully charged capacitor? What is the effective resistance of a fully charged capacitor to current flow?

The behaviour of a capacitor as a circuit element when uncharged or fully charged is useful for a quick analysis of a circuit:

What is the voltage drop across an uncharged capacitor? The answer is zero, from the formula  $V=q/C$  with  $q=0$ . An uncharged capacitor therefore offers no effective resistance to direct current flow.

How much current is flowing across a fully charged capacitor? The answer is zero, whatever the driving potential. So the effective resistance of a fully charged capacitor is infinite.

## Capacitors in Series and Parallel

This section shows the total equivalence of capacitors when in series and in parallel. In series we sum the voltages and hence the overall capacitance is the harmonic mean. In parallel we sum the charges – because the two plates are just one big capacitor – so the capacitances sum.

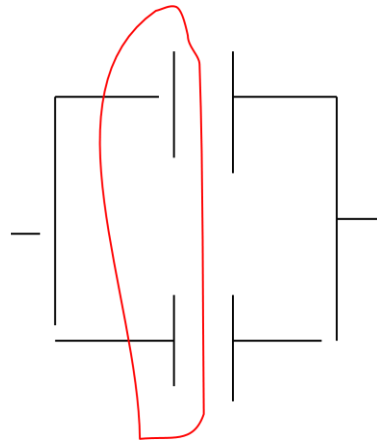
Note how you remember the proofs not just the formulae. Students will tend to remember “capacitors are the opposite of resistors.” This, on its own, simply adds to the apparently meaningless jumble of physics facts!

Series:

$$\begin{array}{ccccccc}
 +q & & -q & & +q & & -q \\
 | & & | & & | & & | \\
 \text{---} & & \text{---} & & \text{---} & & \text{---} \\
 & & & & & & \\
 V_1 & + & & & V_2 & = & V \\
 q/C_1 & + & & & q/C_2 & = & q/C
 \end{array}$$

Parallel:

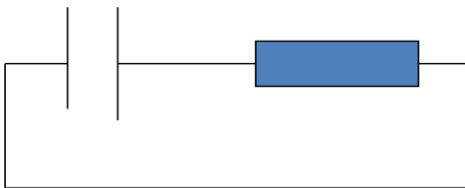
$$q_1 + q_2 = q$$



$$\text{So } C_1V + C_2V = CV$$

$$\text{and } C_1 + C_2 = C$$

## Discharging a Capacitor



PD across  $R$  is  $q/C$

Therefore current is  $I=q/CR$

Therefore discharge time scale =  $CR$

Let's return to the defibrillator problem. One of our learning issues was to find out about a pulse timescale. The capacitor holds the clue to this. Consider a capacitor discharging through a resistor  $R$ . Potential difference across  $R$  is  $q/C$ , the charge on the capacitor divided by its capacitance. Therefore current is the potential difference divided by  $R$  or  $I=q/CR$ . Comparing this with  $I = dq/dt$  we see that  $CR$  is a time. The only time involved in this problem is the timescale to discharge the capacitor. This must therefore be of the order of  $CR$ . To get it exactly we use the exact relation between a time varying current and charge,  $I=dq/dt$ . This gives equation (1). Integrating this equation gives (2) – you can simply verify this by differentiating (2) to show it satisfies (1). We have fixed the constant  $q_0$  to be the charge at time  $t=0$ .

$$dq/dt = -q/CR \tag{1}$$

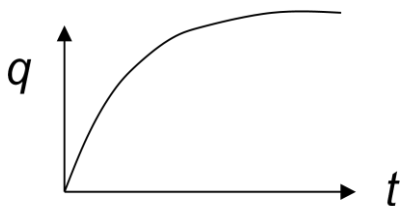
$$q = q_0 \exp(-t/CR) \tag{2}$$

So we see the meaning of the timescale  $CR$  – it is the time taken for the initial charge on the capacitor to fall to  $1/e$  of its original value. Incidentally, if  $R$  is in ohms and  $C$  in Farads,  $CR$  really is in seconds!

Just out of interest, what about the time taken to charge a capacitor? We can think of the charging process as discharging charge of the opposite sign, so we might expect  $q = -q_0 \exp(-t/CR)$ . However, we don't start from charge  $-q_0$  but from zero. So we have to add an integration constant  $q_0$ . This gives us (3).

Charging = discharge –ve charge so expect  $q$   
 $= -q_0 \exp(-t/CR)$  – but start from zero not  $-q_0$   
 so add  $q_0$ .

$$\text{Hence } q = q_0(1 - \exp(-t/CR)) \quad (3)$$



## Defibrillator Problem

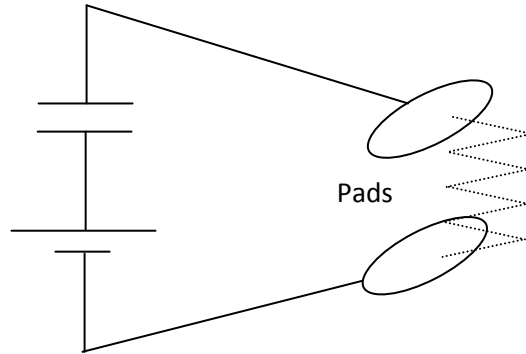
We can now apply what we have found out to the defibrillator problem. A simplified circuit must contain a capacitor and a battery to charge it up. The duration of the pulse must be the time constant of the  $CR$  circuit. We can assume the internal resistance of the circuit is small in order that the energy is dissipated in the chest. Then, since  $R$  is given, we can calculate  $C$ . Finally, we know how much energy we want a pulse to deliver, so we can calculate the required voltage. The circuit in a real defibrillator would have to be more complex to deliver this voltage and to prevent the capacitor discharging through the battery.

Duration of pulse  $T = CR$

so  $C = T/R = 20 \times 10^{-3} / 150 = 1.3 \times 10^{-4} \text{ F}$

Energy in the pulse  $= \frac{1}{2} CV^2 = 360 \text{ J}$

So  $V = (720/1.3 \times 10^{-4})^{1/2} = 2350 \text{ V}$ .

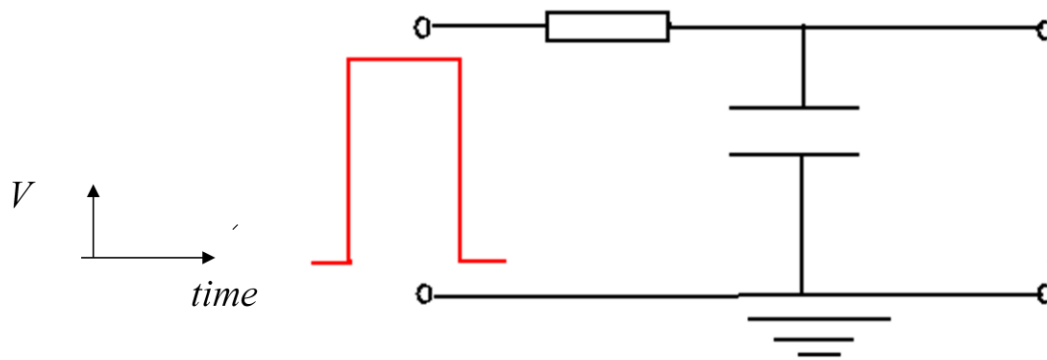


## Summary

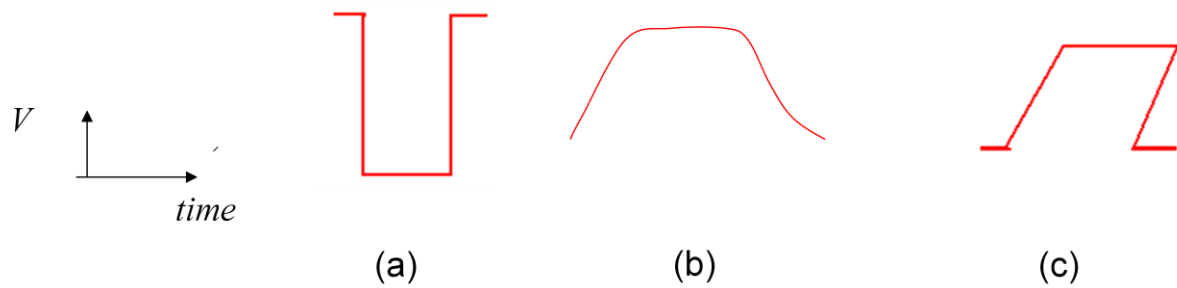
- Charge is accumulated current ( $q=It$  or for non-steady currents  $I = dq/dt$ )
- Ohm's law is  $I=V/R$  ( $1/R$  is the constant of proportionality)
- Real materials as individual current elements are often non-ohmic ( $R$  depends on  $V$ )
- Resistors in series add. Resistance is measured in Ohms (= 1 volt/1amp)
- Capacitors in parallel add
- The pd across a capacitor is  $V = q/C$
- The timescale for discharge of a  $CR$  circuit is  $CR$
- The rate of energy dissipation in a resistor is  $I^2R$  (Joule's Law)
- The rate of doing work in a circuit is  $VI$
- The energy stored in a capacitor is  $\frac{1}{2} q^2/C$

## SAQ

What is the output signal for the circuit shown in the figure for the given input?



Outputs:



The answers appear on the following page

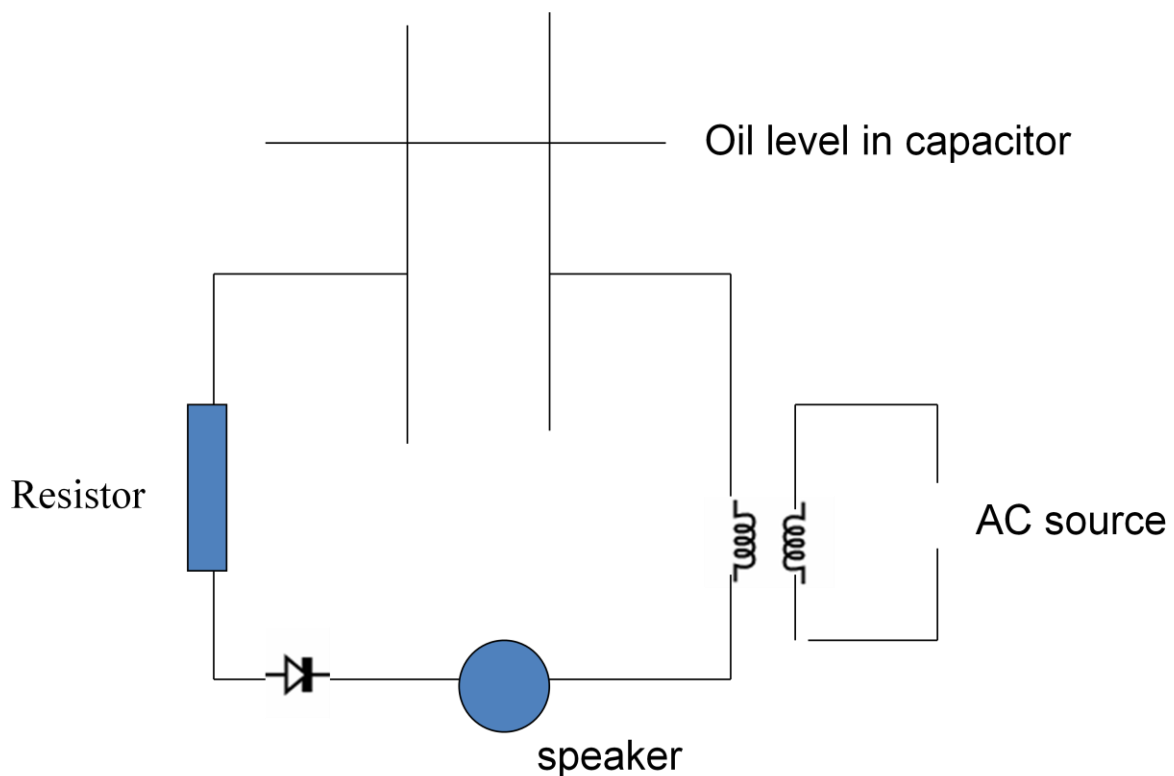
## Answers

- (a) Wrong: perhaps you're thinking that the capacitor charges up and then discharges, but even were that the case it wouldn't invert the potential
- (b) Correct: the capacitor smoothes the initial rise until it is fully charged; once the voltage drops to zero it discharges exponentially.
- (c) Wrong: the initial slope is the right idea, but it has to be smooth. The trailing slope can't occur, because the potential has to be single-valued at any time.

## Second Problem Statements

So far we've dealt with steady currents. We now come to a problem that introduces the behaviour of circuits with a periodic current, Alternating Current or AC circuits.

Problem: The figure shows a proposed device for measuring oil level. As the oil level changes so does the capacitance. At a certain level the speaker sounds as a warning. What values of the circuit elements could be used?




Let's analyse the Learning Issues in the problem.

First of all we need to identify the circuit elements in the figure.

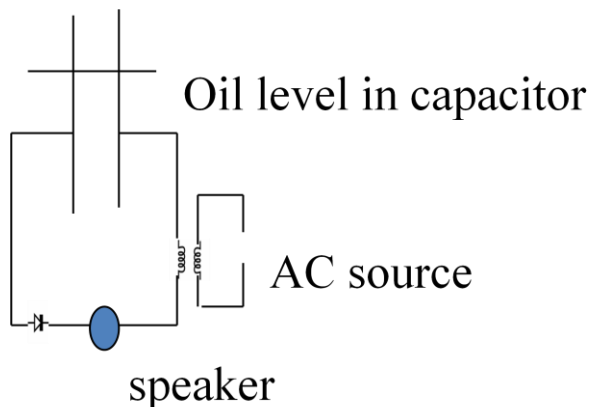
Then we need to think about why the capacitance changes and what affect this will have on the circuit.

## Inductors

 Inductors – the LC circuit



The coil symbol is an Inductor, usually made by winding a wire coil. Inductors store magnetic energy. The stored energy is  $\frac{1}{2}LI^2$ .



As circuit elements, an inductor produces a back emf of  $Ldi/dt$  or  $Ld^2q/dt^2$  where  $L$  is the inductance measured in Henrys.

**Inductance L is measured in Henrys (= volts seconds per amp)**

The pd in an LC circuit is  $Ld^2q/dt^2$  across the inductor +  $q/C$  across the capacitance, which will sum to zero if there are no other elements. Hence in an LC circuit  $d^2q/dt^2 = -(1/LC)q$

We can compare this with the harmonic oscillator equation to see that an LC circuit oscillates harmonically with angular frequency  $\sqrt{1/LC}$ .

Compare this with  $d^2x/dt^2 = -\omega^2 x$ .

*We see that the charge in an LC circuit oscillates harmonically like the pendulum*

**What is the period of an LC circuit? Explain physically what is happening in the oscillating circuit.**

The period is  $2\pi(LC)^{1/2}$ . Physically the current is flowing between the capacitor and the inductor. As the capacitor charges the emf builds up until the current flow decreases. At this point the low rate of change of current reduces the back emf from the coil allowing the capacitor to discharge through it. The current builds up increasing the emf in the coil and sending the current back into the capacitor. Thus, electrical energy in the capacitor is continually converted into magnetic energy in the inductor and vice-versa.

## Mechanical analogy

The LC circuit is the analogue of the harmonic oscillator.

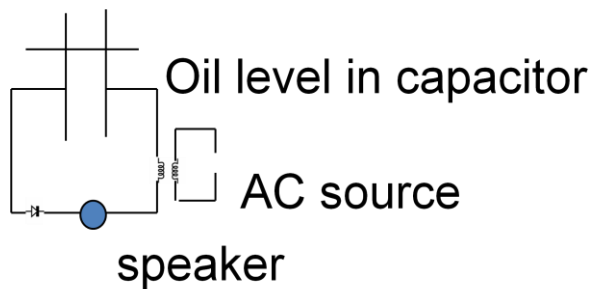
What is the mechanical analogue of an LR circuit?

For an LR series circuit we have

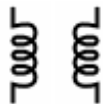
$$L \frac{dI}{dt} = -RI \quad (1)$$

This is the same as the CR circuit except that charge is replaced by current. The current decays away exponentially on a time scale of  $(L/R)$ . The magnetic energy stored is analogous to the kinetic energy of a mass. The dissipation through the resistor is analogous to the dissipation of kinetic energy by friction.

Consider the proposed circuit:



1. What is happening between the main circuit and the AC source?



In practice these coils would be wound around each other. The changing current in one *induces* an emf in the other. Thus the circuit is fed with an alternating voltage. (One could of course connect this directly).

2. What is the purpose of the diode in the circuit?



Before we answer this, let's consider the AC source. What is the frequency of the AC source. Is it audio, so that we can hear it in the speakers? We have an LCR circuit – we can't avoid some resistance – so we can use the mechanical analogy to see how it behaves. It will have a resonant frequency of  $1/(LC)^{1/2}$ . Since we won't want to use large capacitances (which would

have to be physically large size) the frequency is likely to be quite high. We therefore do not want to be restricted to audio frequencies. Instead we use a carrier wave at kHz modulated at a few 100 Hz to give a note clearly in the audible range. The idea is that the buzzer should sound when resonance is reached by the effect on the capacitance of a declining sugar level.

Without the rectifier the loudspeaker would attempt to follow the carrier wave. Its inertia would prevent this, so it would remain silent. With the rectifier, the speaker still cannot follow the carrier wave in detail, but it can now follow the average level over the period of the audio period. Thus, the speaker will sound provided that the circuit is near enough resonance to pick up enough energy.

## Capacitance with Dielectrics

A capacitor of plate area  $A$  distance  $d$  apart filled with a dielectric of dielectric constant  $\epsilon$  has a capacitance  $\epsilon A/d$ .

### Look up the dielectric constant of oil

The dielectric constant of benzene is 2.3 – you may have something different. Use your data.

Use the parallel capacitor formula to work out the capacitance as the oil level changes. From you analogy with driven damped HO – what is the behaviour of the circuit?

The LC circuit resonates at a frequency  $(1/LC)^{1/2}$ , thus only when  $C$  has a particular value. The circuit can be used to indicate when the oil level drops through this level. Why is this not a good warning device if a minimum oil level is critical? Why have we ignored the resistance? What effect would adding another resistance have? To answer this, go back to the comparison with the damped harmonic oscillator. Would adding a resistance help? We'll give you the answer in the next section.

## Solution of the circuit problem and summary

So summarizing where we've got to so far:

The circuit is supplied with a carrier wave at a frequency at which it can resonate with reasonable values of C and L. This is modulated at an audio frequency to give an audible warning.

The rectifier effectively filters out the carrier frequency to allow the energy to be transferred to the speaker coil. (In effect impedance matching the systems to the speaker.)

The capacitance changes as the oil level drops since it is effectively two capacitors in parallel

Inserting a resistance in the circuit broadens the resonance so the signal can be heard over a range of capacitance values (i.e. a range of oil levels). The circuit is an exact electrical analogue of the driven damped harmonic oscillator

Use session 3 on harmonic oscillators to complete the problem.

## SAQs

1. Why does switching off a high current circuit produce a spark
  - a) Because of the inductance in the circuit
  - b) Because of the capacitance in the circuit
  - c) Because a high current implies a high voltage which generates a spark
  - d) Because of the inertia of the current which means it cannot stop flowing immediately
2. What changes would you make to increase the resonant frequency of an LCR circuit?
  - a) Increase  $L$
  - b) Increase  $C$
  - c) Increase  $R$
  - d) Decrease  $L$
  - e) Decrease  $C$
  - f) Decrease  $R$

The answers appear on the following page

## Answers

1.

- a) Correct:  $V$  can be high if  $I$  changes suddenly, unlike the alternatives which cannot generate a  $V$  higher than the supply voltage.
- b) Wrong: The capacitance cannot generate a higher voltage than the one used to charge it.
- c) A high current does not necessarily imply a high voltage – put the other way round, you can get very large currents by shorting a low voltage battery.
- d) Wrong: Another example of the “flow” analogy letting us down.

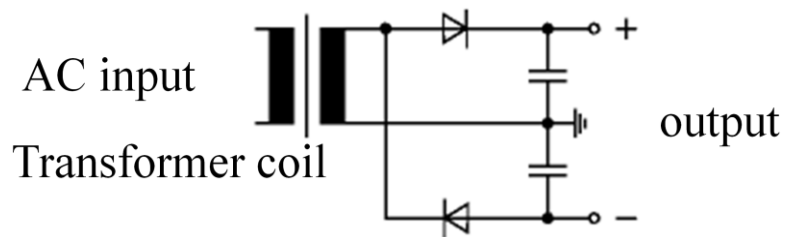
2.

We have  $\omega_0 = \frac{1}{\sqrt{LC}}$ , so to increase the resonant frequency,  $\sqrt{LC}$  must decrease. Therefore the correct answers are d) and e). The resistance value does not affect the resonance frequency.

# Additional Problems

## Voltage Doubling

The Cockcroft-Walton generator was used to produce high voltages in the first splitting of the atom, although it was actually invented somewhat earlier by the Swiss physicist Heinrich Greinacher



Image<sup>4</sup>

How does the circuit produce a DC voltage at output that is doubling the peak to peak AC voltage at input? How is this consistent with conservation of energy?

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<sup>4</sup> Massive Cockcroft-Walton, hackerfriendly's photostream as posted on [www.flickr.com](http://www.flickr.com), Creative Commons Licensed

Answer:

To understand how the circuit produces a doubling of the peak voltage at output, consider what each rectifier does and what each capacitor does to each half of the AC waveform. On one half of the cycle one capacitor will charge up, while on the other half of the cycle the other charges. The rectifiers stop each capacitor discharging through the transformer. Combined in series the potentials of the capacitor add to give double the input voltage. To reconcile this with conservation of energy, the current is reduced by half. This is because the two capacitors in series have half the capacitance of each alone, thereby double the discharge timescale CR.

## Problem 2

A CR series circuit is connected to a battery, voltage  $V$ , by closing a switch. Once the connection is made what is the energy delivered by the battery in charging the capacitor? Compare this with the energy stored in the capacitor when it is fully charged to the voltage  $V$ . Where is the missing energy?

$$V_b = IR + \frac{q}{C} \quad (1)$$

$$\text{So} \quad \int RI^2 dt = \int V_b Idt - \int \frac{q}{C} \frac{dq}{dt} dt \quad (2)$$

$$\begin{aligned} \text{Hence} \quad \int RI^2 dt &= \int V_b dq - \int \frac{q}{C} dq \\ &= V_b q - \frac{1}{2} \frac{q^2}{C} \quad (3) \end{aligned}$$

$$= \frac{1}{2} V_b q \quad (4)$$



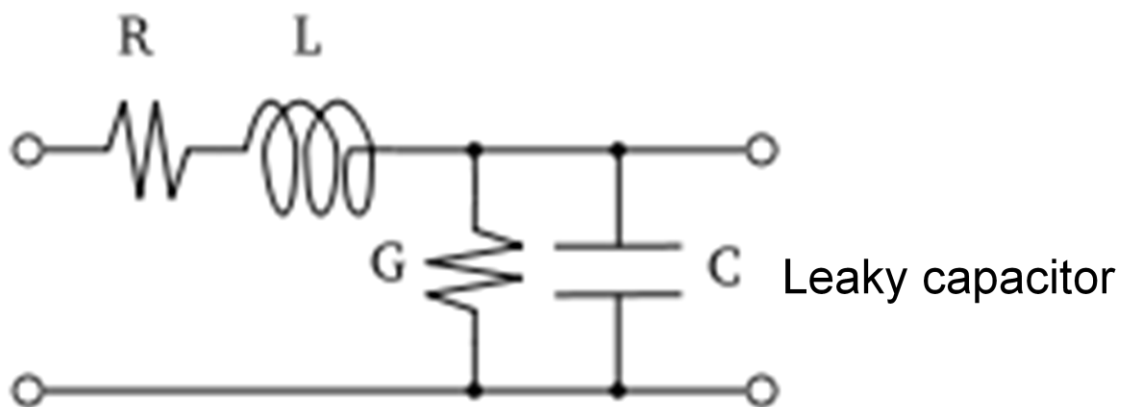
Answer:

There are two circuit elements providing the potential drop across the battery. The sum of the potential drop across each is the battery voltage. This gives us equation (1). Note the subscript 'b' on  $V_b$  to remind us that this is the constant battery voltage. Re-arranging and multiplying by  $I$  we get the energy dissipated in the resistor in terms of the energy supplied by the battery and the energy stored in the capacitor, equation (2). Integrating the right hand side, remembering that  $V_b$  is constant, gives (3). Now in the final state there is no current, so there is no voltage drop across the resistor. This means that  $q=CV_b$ . So we can put this into the final term. This tells us that half the energy supplied by the battery is dissipated in the resistance.

A paradox arises if  $R$  is zero. The energy supplied by the battery is still twice that stored in the capacitor, because the terms on the right hand side of the equation are unchanged. But there is now no resistance to dissipate that energy. Where does it go?

### Problem 3

Given the (somewhat oversimplified) equivalent circuit for a transmission line, work out the limitations on the time it would take to transmit a binary signal.



Element of a transmission line

What does a square wave input look like at output?

The circuit shown represents an element of a transmission line – as an approximation we can think of it as lumping the whole line together into an inductor, leaky capacitor and resistor. You might like to think how we know from the circuit diagram that the capacitor in the circuit is leaky.

Its charge leakage from the capacitor is represented by the resistance  $G$ .

The object of the exercise is to determine what happens to a square wave pulse (e.g a binary digit sent down the line.) What limitations will this impose on speed of transmission of a signal? This is similar to self-assessment question at the end of section 2. Here however, the inductor responds to the sudden increase in current at the leading edge of the pulse by creating a back emf which slows the rise in the signal. The output signal will therefore start with an exponential rise, before flattening out to the top hat shape. The trailing edge will be smoothed by the capacitor. The additional complication of the leaky capacitor means that signal output will have a lower amplitude than the input.

Looked at another way, the circuit filters out the high frequency components which arise from the sharp edges in the input signal– the resonance curve of the circuit imposes a transmission factor on the Fourier components of the incoming oscillation.

Visit the web site <http://phet.colorado.edu/web-pages/simulations-base.html>

## Meta tags

Author: Samuel Atarah.

Owner: University of Leicester

Title: Enhancing Physics Knowledge for Teaching – Current, Electricity

Keywords: Capacitors; Current; Electricity; Inductors; Resistors; sfsoer; ukoer

Description: In this session we will be introducing electric currents. We will consider how an electric current behaves in DC and AC circuits, and how these circuits are described with circuit components like resistors, capacitors and inductors.

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Language: English

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## Additional Information

This pack is the Version 1.0 release of the module. Additional information can be obtained by contacting the Centre for Interdisciplinary Science at the University of Leicester. <http://www.le.ac.uk/iscience>

