# Session 5

# Electric and Magnetic Fields

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# Contents

# Welcome

Welcome to session 5 of the Physics programme. This session will introduce you to electric and magnetic fields. We'll look at what we mean by a field and at what electric charge is and relate the two through Gauss's theorem. From this we'll derive Coulomb's law for the force between charges. Then we'll look at the concept of electrical potential which is related to the work done in moving a charge through a field. We'll return to the notion of capacitance which we used in session 4, this time looking at how capacitance is computed. Finally we'll take out first look at magnetism from the point of view of the magnetic effects of current.

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Session Editor – Tim Puchtler

# Learning Objectives

- Recognise charge as a measure of the strength of interaction
- Define the flux of a field and apply this to electric fields
- Use Coulomb's law to compute the forces between charges
- Define electrical potential
- Derive capacitance and the effect of dielectric materials
- Describe the force between current carry wires

# The Problem

As usual we'll begin with a problem that will cover the main ideas. The problem will involve overhead power cables. We'll present it in two parts.

**Pepco Lineman Dies of Injury, Tuesday, October 05, 2004:**
For Immediate Release:

*October 5, 2004 Michael Boxley, 47, a veteran Pepco overhead lineman, died late Monday in Washington Hospital Center from injuries sustained during an on-the-job accident that occurred Sunday, the company announced today. "We extend our deepest sympathy and prayers of comfort to Mike's family," said Pepco President Bill Sim. "Mike was a senior member of our line mechanic team who was dedicated to keeping the lights on for our customers. His death is a terrible loss not only to his family but to our entire company." Mike was assigned to the company's Forestville Service Center in Prince George's County. He was completing repairs atop a 13,000-volt distribution pole in a residential neighborhood in Landover, Md., when he came in contact with a live wire. An assistant on the ground called for help immediately. Fire and rescue personnel along with coworkers from Forestville came to Mike's aid and he was flown to the hospital where doctors performed surgery in an effort to save his life. The accident remains under investigation.*

In fact, back in the 1890s, when electrical power was new and this field was just emerging, more than half of all line workers died on the job.

Image[1]

[1] Helicopter picking up a linesman, b0jangles' photostream on www.flickr.com, Creative Commons Licence

EPRI Journal, Spring 2007:

*Live work requires extraordinary attention to worker safety, especially in aerial work, where the combination of high-voltage wires and hovering helicopters offers scant margin for error," says PSE&G's Tom Verdecchio. "Using the EPRI Lenox Center to perform electrical testing of the new helicopter platform and live work procedures has helped us increase worker safety and improve our transmission line maintenance*

So the first problem we want to investigate is: How can linesmen work safely on live wires? The second part of the problem, which we'll come back to later, involves the cancer risks of the magnetic fields from these cables.

Let's look at the learning issues: How is it that one worker died when contacting a live wire whereas others routinely operate on live cables? One answer is insulation. Is that enough? Consider birds on wire?

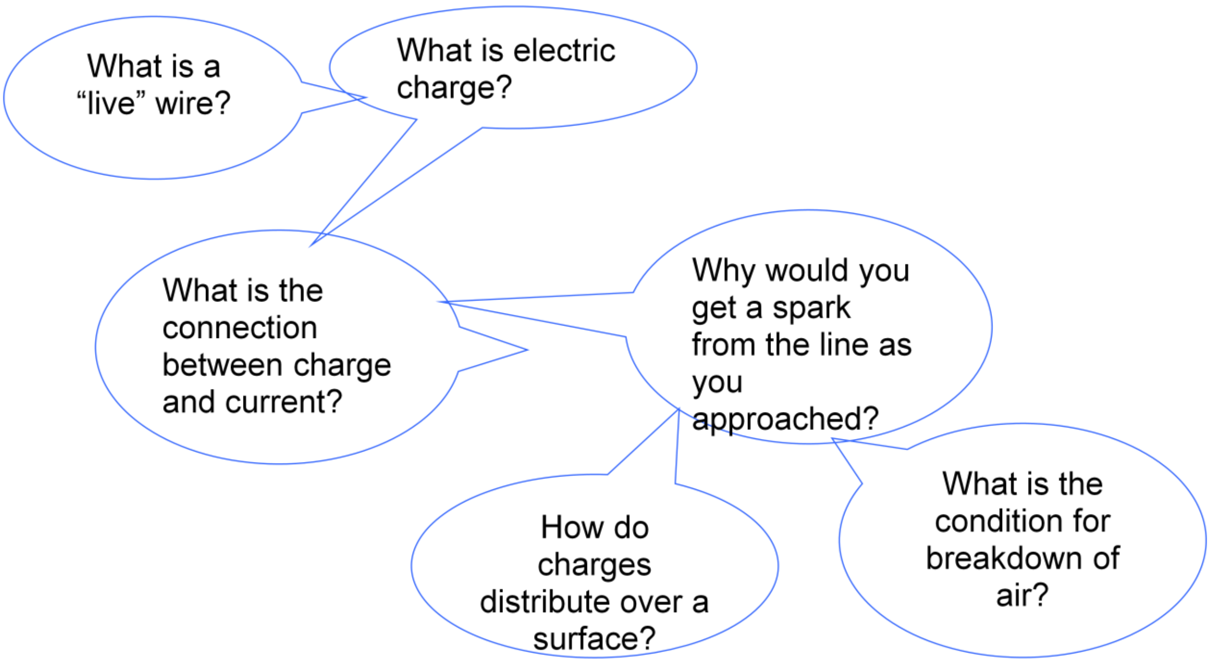
Additional issues: what are the other potential dangers from overhead power lines: cancers? We will come back to this.

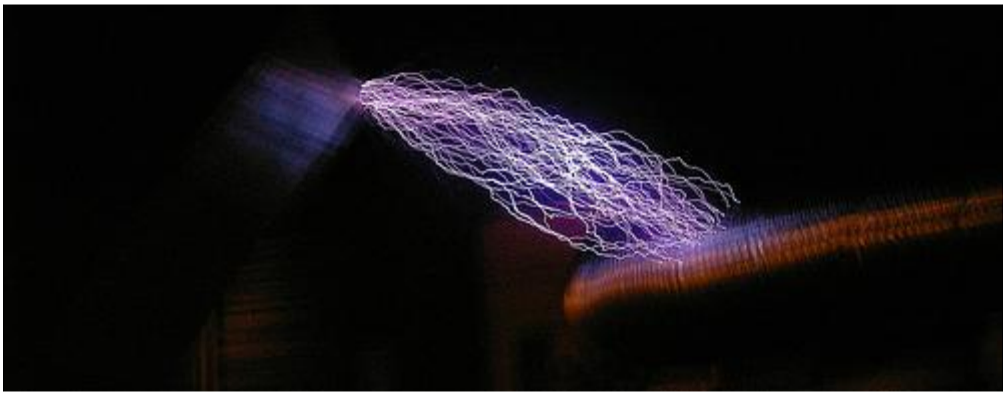
# Electric Fields

This first section will deal with electric charges and the electric fields they generate. We'll then go on to relate fields to potentials which will enable us to see how a linesman can be protected.

**Draw a concept map of the questions you might ask about high tension cables**

The following is an example concept map. It will no doubt be different to yours, but hopefully the main points should be agreed upon.



Image[2]

[2] Sparks, hackerfriendly's photostream, from www.flickr.com, Creative Commons Licensed.

# What is the connection between charge and current?

**Rub a plastic ruler on a woolly jumper (or cat's fur). Note the effect of the ruler on your hair or some small pieces of paper.**

On a somewhat larger scale friction on the rubber belt of a van de Graff machine produces charges which can be used to drive a current in a circuit. Therefore, current is flow of charge. The units of charge are coulombs, defined as the charge accumulated by a current of 1 amp flowing for one second.

Charging a Van de Graff machine will produce a current in a circuit.

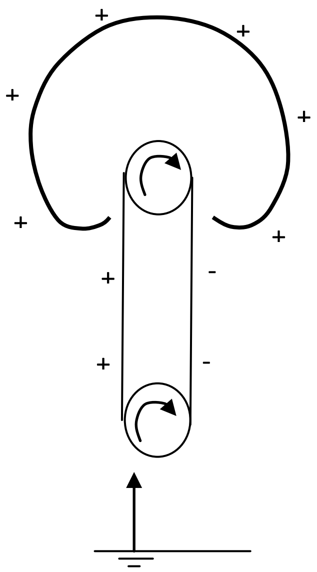
Units: Charge: Coulombs

Current: Amps = Coulombs /sec

**Find out how a coulomb is defined.**

**What pedagogically do you think of the definition?**

Note that when currents started to be investigated around 1800 they were initially thought to be distinct from charges. So people studied four kinds of electricity: positive and negative charges and positive and negative currents. The connection isn't therefore obvious to anyone who doesn't have an intuitive picture of a charge carrier, such as an electron.



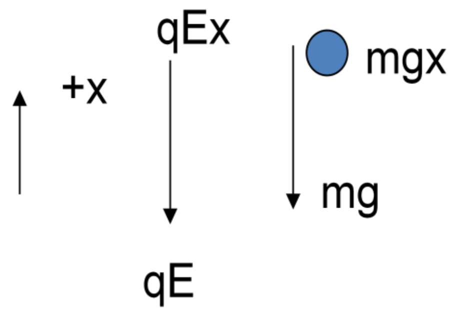
# What is Charge?

So we come back to the question: What is charge? There is a fundamental answer to this, the understanding of which removes a lot of confusion.

Let's return to our idea that a physical theory is defined by the expression for energy of the system, made up of Kinetic energy and potential energy. Recall our example of the gravitational Potential Energy – mgx. This contains a coordinate (x) defining the state of the system; a field g and a strength of coupling (m) of the field g to the system. The larger m is, the stronger the force of gravity, mg.

This is the prescription we use to construct a Potential energy for any field (invented or real!) In the case of the electric field E, we introduce a coupling constant, lower case q, to get a potential energy qEx. There's a corresponding vector expression if we are dealing with more than one dimension. Note that we have to chose a zero point for the potential energy, but this can be arbitrary – it's choice, once fixed, doesn't affect the physics

In one dimension, the force on a point charge is then, as usual, -d/dx(qEx) or qE;




**The electric field is the force on a unit +ve charge**

Charge therefore appears in two roles - as a quantity giving the strength of the field and as a coupling constant, giving the strength of the interaction between a point charge and the field. In fact there is a third role:

Since the Potential Energy involves both the field E and the coordinate x this tells us that the field affects a charge but also that a charge creates an electric field – and since the charge q is the same in both cases, active and passive charges are equal. This therefore embodies Newton's third law.

The field will also have its own energy which we have to postulate. For the electric field the energy per unit volume of space is $\frac{1}{2} \varepsilon_0 E^2$, where $\varepsilon_0$ is a constant depending on the units used. This is $\frac{1}{2} \varepsilon_0 E^2$ per unit volume.

If E is measured in Volts/metre then $\varepsilon_0 = 8.854 \times 10^{-12}$ farad / metre

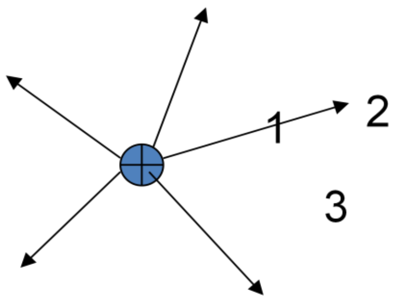
The field then has as its source the distribution of charge.

**The Faraday is defined as the charge carried by a mole of electrons? How many Coulombs are there in a Faraday?**

## Visualizing a field

We have seen that the electric field is analogous in many ways to the gravitational field. We don't feel much of a need to visualise gravity because we can feel it. In our experience the gravitational field is constant, so one arrow attached to a mass is enough to provide a picture. We usually visualise a field that changes from point to point in space by a set of arrows representing the forces on unit test charges placed in the field. Joining the arrows gives the lines of field (or lines of force). This can lead to a number of misconceptions given as examples.

Field of a + point charge. Conventionally field lines start on positive charges and end on negative charges




Where is the field strongest? The following choices represent some common misconceptions

1. Because it is on a field line
2. Because it is at the tip of an arrow
3. Where the field lines are most widely spaced

In order to consider where the field is strongest, we should introduce the idea of the flux of a field.

# Flux

We come now to an important concept that we shall need: that of the flux of a field. The flux of a field through an area is the intensity of the field times the area. We now give a couple of examples:

1: Flux of rain on an umbrella is the volume per second (intensity of the rain) x (cross-sectional) area of umbrella

2: Flux of sunlight through a window is the normal component of energy flow per second through the window (the intensity integrated over all directions times the area)

Flux is conserved if there are no sources or sinks – so it doesn't matter where you measure it. In the case of the illustration, a bucket the size of the umbrella would do equally well.

This is how flux is used in physics – with this *precise* meaning, not a vague term in English. Many students find difficulty appreciating the difference! Even many university students will use physics terms casually, saying for example force when they mean pressure and energy when they mean power. This makes intelligent communication impossible.

**The rain falling on a roof runs off down a drainpipe. What can you say about the flux of water on the roof and down the drain. If there are no sources or sinks in between does it matter where you measure the flux?**

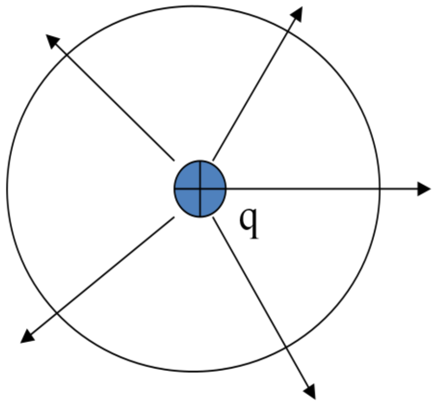
# How electric charge creates electric fields

Gauss's theorem is extremely useful and often the source of some difficulty because it's frequently hidden behind some mathematical symbolism. Gauss's theorem says that:

"The flux of the electric field through a closed surface is equal to the enclosed charge."

There's actually a constant factor that appears in this relation that depends on the units. With charge measured in Coulombs and electric filed in volts per meter the constant is just $1/\varepsilon_0$, the same $\varepsilon_0$ that we met in the expression for the energy of the electric field.

The theorem applies to any surface and any distribution of charge, but to apply it in the general case requires some mathematical dexterity. However, for simple symmetrical situations, such as a charge at the centre of a sphere the application is straightforward and leads to an important result.

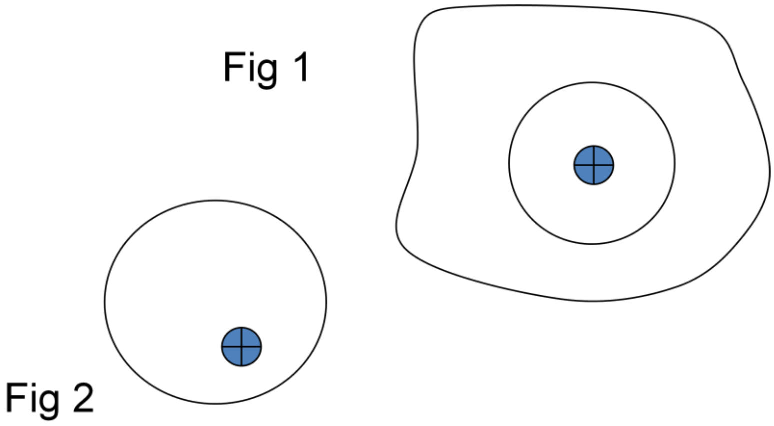



Because the field is constant over the sphere, and pointing normal to the sphere, the flux of E through the sphere is E times the Area.

This equals the enclosed charge q over $\varepsilon_0$.Thus, the field of a point charge q at distance r is $q/4\pi r^2\varepsilon_0$. The force between two charges $q_1$ and $q_2$ is $q_1 q_2 /4\pi r^2 \varepsilon_0$. This is the inverse square law for the electrostatic field analogous to the inverse square law for gravity. It is valid for static charges, and is called Coulombs law.

# SAQs

1. What is the relation between the flux through the two surfaces in figure 1?




a) the same
b) smaller through the outer surface
c) smaller through the inner surface

2. What is the flux through the surface in figure 2 enclosing a charge q?
a) can't tell
b) $q/\varepsilon_0$
c) $> q/\varepsilon_0$
d) $< q/\varepsilon_0$
3. If the distance between two unequal charges is doubled the force on the larger charge is multiplied by
a) 2
b) ½
c) ¼
d) 1 (i.e unchanged)

**The answers appear on the following page**

# Answers

1. a) Correct: there are no sources between the two surfaces so the flux is conserved

   b) Wrong: T The field at the outer surface is smaller, but the area is larger such that the product is unchanged

   c) Wrong: the area of the inner surface is smaller, but the field is stronger such that the product is unchanged.
2. a) Wrong: a direct calculation of the field integrated over the area is difficult, but Gauss's theorem circumvents that: the answer is (b) from the theorem.

   b) Correct: Gauss's theorem tells us that the flux depends only on the enclosed charge not on its distribution.

   c) Wrong: the increase in field due to the proximity of the charge to one part of the surface is cancelled by the increased distance to the remainder.

   d) Wrong: the decrease in field due to the increased distance from one part of the surface is cancelled by the greater proximity to the remainder.
3. c) Correct: the force is given by inverse square law, so the force is proportional to $1/r^2$

   a), b), d): Wrong: the force is given by inverse square law, so the force is proportional to $1/r^2$

# Fields and Potentials

Having discussed the idea of a field in the previous section we'll turn here to potential. This will allow us to address some of the issues raised by the linesman problem.

## Fields and potentials

Let's begin with our old friend, the potential energy. Recall that it's the work done on moving a charge through a distance from some reference point. The force on a charge q is qE so the work done is qEx if the field is constant, or the integral of qE dx if the field is a function of position.

$$\text{Work done} = \text{Force x distance} = \quad qEx = \int qEdx \qquad (1)$$

If we move a stationary particle from one position to another in a system, the work done in moving the particle must equal the change in potential energy of the particle.

$$PE = -WorkDo\imath$$

The potential $\phi$ of a field at a point is defined as the potential energy of a unit charge. Conversely the field is the rate of change of potential with respect to position. So if we think of high and low potential as a landscape, the field at any point is the slope of the land. Often the symbol V is used for potential, as we did in session 3.

Potential $\phi$= PE of a unit change

Of course, knowing the potential we can recover the potential energy of any charge at a location in the field – it's just q$\phi$; for a charge density ρ per unit volume, the field energy per unit volume would be ρ$\phi$.

$E = -d\phi / dx$

Therefore energy of a charge $q$ is $q\phi$

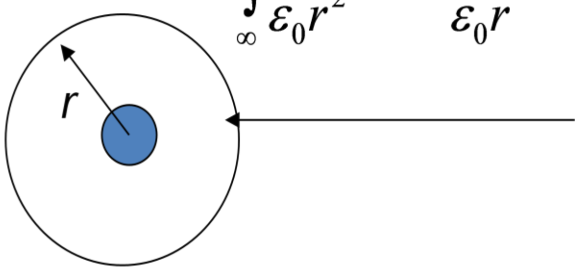
Energy per unit volume of a charge density is $\rho\,\phi$

What is the potential of a point charge q?

To solve this we go back to the definition – the only thing we know about potential is that it's the work done in moving a unit charge from a reference point to the point at which the potential is required. A sensible reference point here is $\phi = 0$ at infinity.

So to get the potential we integrate Edr = $(q/\varepsilon_0 r^2)dr$ from infinity along a radial path to a point at radius r from the charge . So the potential of a point charge at distance r is $q/\varepsilon_0 r$

$$-\int_{\infty}^{r} \frac{q}{\varepsilon_0 r^2} dr = \frac{q}{\varepsilon_0 r}$$




To answer the final question numerically you'll need to look up the charge on the electron. The work done is called the electron volt and the answer to the question gives the conversion factor between electron volts and Joules.

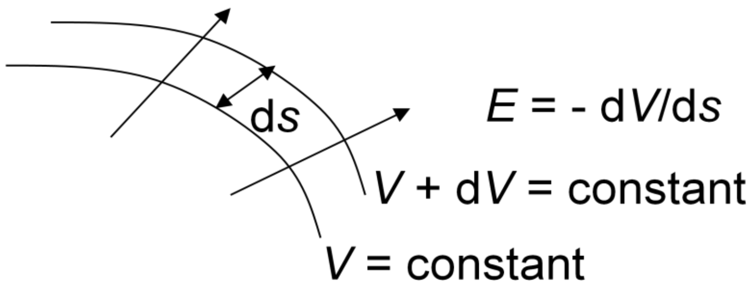
**How much work is done (in Joules) in moving an electron through 1 Volt?**

## Equipotentials

In the linesman problem we need to look at conducting surfaces. What can we say about these based on what we have learnt so far?

We've seen the field is the derivative (or gradient) of the potential i.e. it flows down the slope in the landscape formed by the highs and lows of the potential. The slope *along* an equipotential is zero, so the field is therefore perpendicular to surfaces of equal potential.

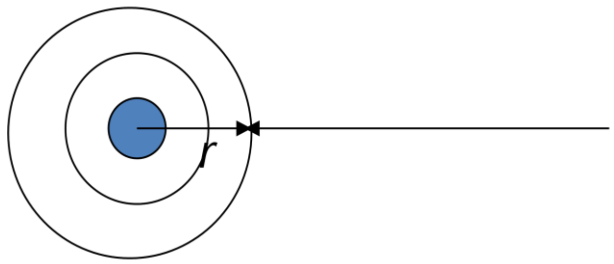
Now, conducting surfaces are equipotentials. Why? – Because the charge flows on them freely to eliminate any forces, hence to eliminate any gradients of potential. No gradient means equipotential.




**Fields are perpendicular to surfaces of equal potential.**

**Conducting surfaces are equipotentials!**

What is the potential of a conducting sphere of radius $a$ carrying a total charge $q$?




What is the potential of a sphere of radius a with charge q at a distance r? This is a good revision question: We're going to need to integrate the field from infinity to r, just as we did for the point charge. To get the field we apply Gauss's theorem: the field outside a sphere with charge q is the same as that of a point charge q. Hence the integral to r is the same as that for a point charge, namely $q/4\pi\varepsilon_0 r$. This is true for any radial ray, so the sphere is an equipotential. At the surface of the conducting sphere of radius a, the potential will be $q/4\pi\varepsilon_0 a$.

# Faraday Cage

We are now finally ready to return to the linesman problem.

The lineworker wears a protective suit and uses insulated tools or works from a Faraday cage:

Images[3]

We can therefore address the questions of how he avoids electrocution by looking at the field inside a conducting sphere. By Gauss's theorem if the charge inside the sphere is zero then the field is also zero. External electric fields do not penetrate conducting surfaces. How can this work? The key is the word conducting: the charges are mobile so assemble themselves to neutralise the field.

Now our cage and lineworker are not spherical. So how does this result depend on shape of the conducting surface? By conservation of flux it is clear that the shape doesn't matter. The flux through any surface that does not enclose a charge is zero.

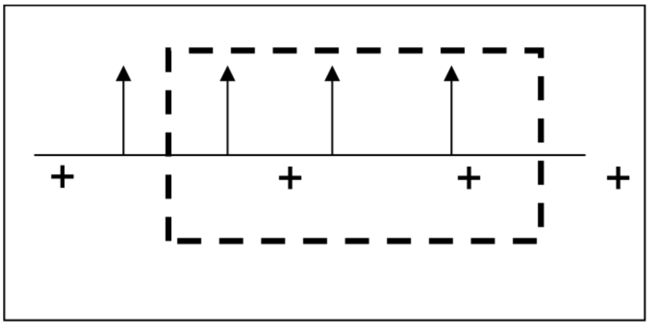
We conclude that unless the lineworker provides part of a closed circuit for a current to flow through, he or she remains safely in a zero electric field inside the conducting suit or cage.

There's one more thing to explain at this point – that's the sparking.

[3] Damaged Safety Gloves, NIOSH, and Tesla's Sparks, Spleeney, obtained from www.flickr.com, Creative Commens Licensed.

## Sparking

Why do we get sparking from sharp pointed objects as the lineworker approaches the line? We need to find out the field near a surface that is not a sphere. Now any surface looked at on a small enough scale is flat. So let's find the field above an infinite flat surface with a charge per unit area, or charge density, σ.




Of course, we use Gauss's theorem. Consider the closed surface represented by the cross section shown as the dotted line above the infinite plate carrying the positive charges. The flux inside the conductor is zero as we've just seen. The flux through the top surface therefore equals the enclosed charge times $1/\varepsilon_0$. Since flux is field times area, the field is just the charge per unit area times $1/\varepsilon_0$ or $\sigma/\varepsilon_0$

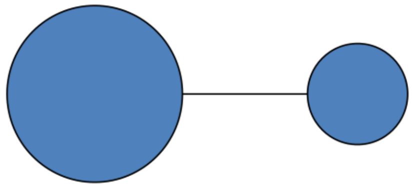
Gauss's law:

Flux = charge

Field = charge /area

$E = \sigma/\varepsilon_0$

You might guess that the charge would move away from a sharp part of the conductor thereby lowering the field. Only a calculation will decide: to model the different radii of curvature, consider two spheres of different radii at the same potential. Which has the larger charge? Well, suppose we have a charge capital Q on the larger sphere and lower case q on the smaller. The potential of each is essentially the respective Q/R. If the spheres are at the same potential these must be equal. But we want to express this in terms of surface density of charge, σ: so we substitute for each charge q the corresponding $4\pi r^2\sigma$. This tells us that σr is a constant, or that the surface charge density is proportional to 1/ (the radius). The surface charge density gets bigger as the radius goes down.



$$V = Q/4\pi\varepsilon_0 R \quad = \quad q/4\pi\varepsilon_0 r$$

But $Q = 4\pi r^2 \Sigma$ and $q = 4\pi r^2 \sigma$

So $\sigma / \Sigma = R/r$ The charge density increases as the radius goes down.

So we now know that the surface charge accumulates towards a sharp part of the surface. What does this do to the field? Well, we saw previously that, up to a constant, the field equals the surface charge density. So at a sharp edge the field is strongest.

Increased charge density => increased field near a sharp point

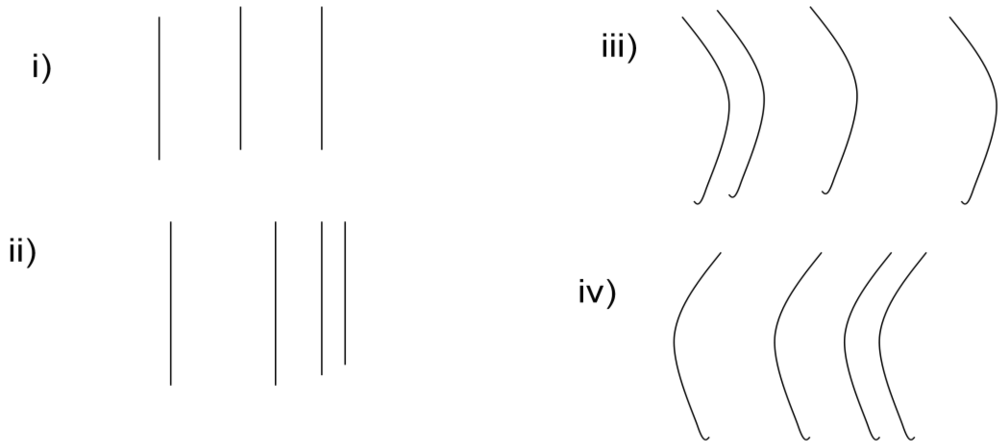
Can we use this to explain the sparking? A high electric field strips air molecules of electrons so the air becomes conducting. The current contributes to the removal of electrons from more atoms. Finally recombination of electrons and atoms produces optical emission.

Are we satisfied with this account? Think about the current flow for a moment. -- This explanation holds only if there is a complete circuit! But we do see sparking. Where is the fallacy? – It's in our assumption that we can consider a DC line. Really we should be considering the effect of Alternating current. We'll turn to that in the next section.

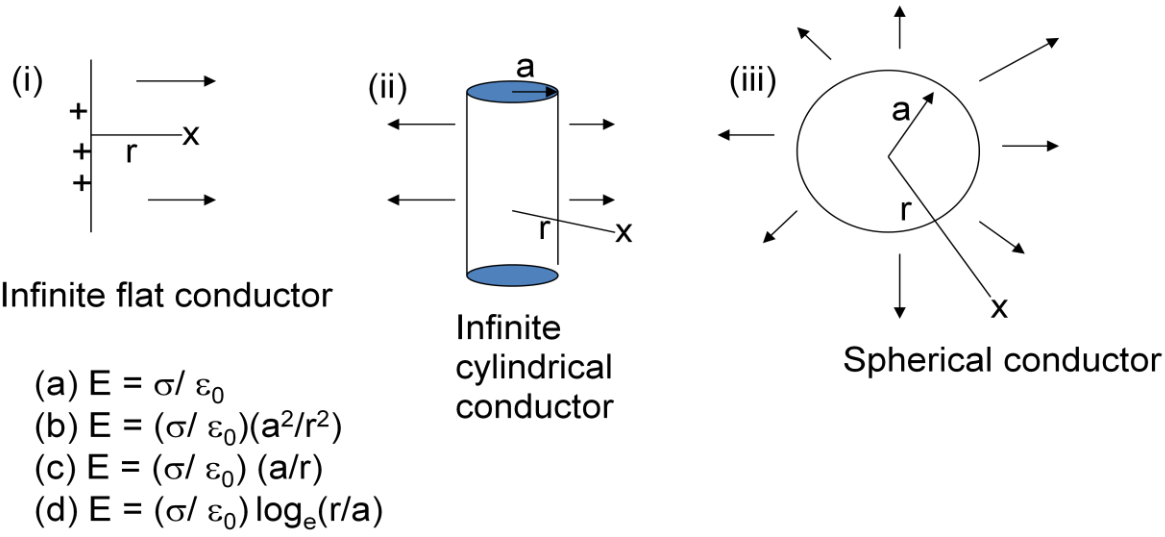
## SAQs

1. In which of the following diagrams of equipotentials is the field
   a) constant
   b) Increasing from left to right
   c) diverging from left to right

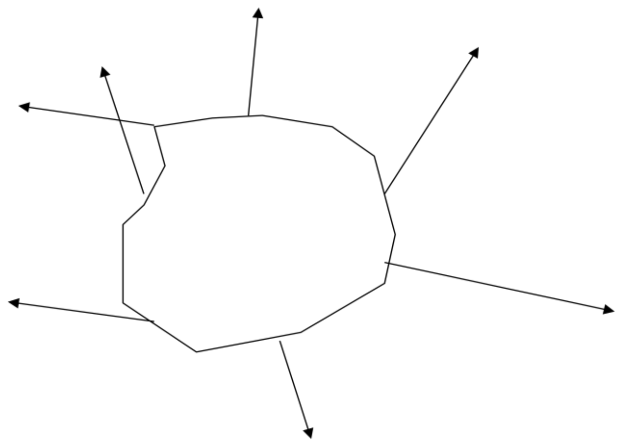
In each case the increment of potential is constant between consecutive surfaces.




2. Fill in the missing number for the potential energy of a charge of 4 C and a charge of 3 C a distance 2m apart. ___ $/\varepsilon_0$ Jules
3. Match the electric fields to the figures




4. What is wrong with this picture of the field lines outside a conducting surface (choose as many as you think fit)?



(a) field lines cannot cross

(b) field lines must be perpendicular to a conducting surface

(c) the field lines should cluster near the sharp area

The answers appear on the following page

# Answers

1. a) Correct answer is i) because the equipotentials are equally spaced.

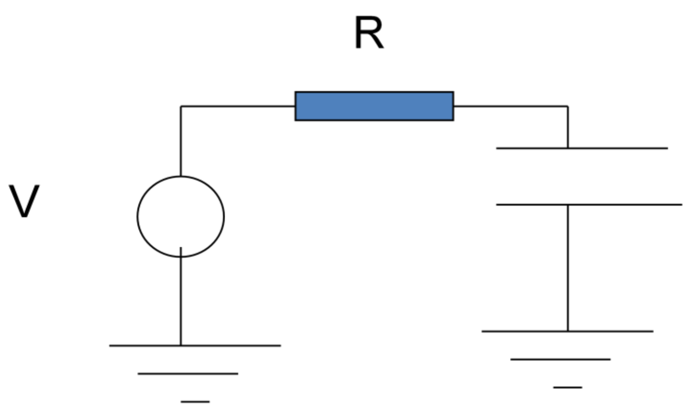
   b) ii) is the correct answer because the equipotentials are getting closer (Think of a contour map and the corresponding slopes of the land).

   c) iii) is correct, which can be seen once you draw the filed lines perpendicular to the equipotentials.
2. Correct answer is 6 ($q_1$ $q_2$/r x 1/ $\varepsilon_0$)
3. ((i) = (a); (ii) = (c); (iii) = (b)
4. All three answers should be chosen.

# Capacitance

In order to understand why sparking occurs between the linesman or his cage and the AC power line; we have to return to the subject of capacitance.

Now consider an AC line. Do we still get sparking even though there is not a complete circuit?




The figure above is a representation of the line and linesman - the linesman is represented by the resistance and the top plate of the capacitor. The lower plate is the Earth or the rest of the Universe. With an AC voltage we don't need a circuit for sparks to fly! – In effect the current can cross the capacitor (although of course no actual charge flows between the capacitor plates).

So we need a geometrical model for the linesman in order to calculate his capacitance.

We'll model the linesman as a sphere. This will give us a correct order of magnitude in a simple fashion. If we know the potential V produced by a charge q on an object it is easy to work out the capacitance – it is just the ratio q/V. We already know the potential of an isolated sphere of radius a – it's $q/4\pi\varepsilon_0 a$ . Since capacitance is q/V we can identify C as $4\pi\varepsilon_0 a$

$$q = CV = Cq/4\pi\varepsilon_0 a \quad \text{so} \quad C = 4\pi\varepsilon_0 a$$

Have a go at estimating the capacitance of the linesman. I'll give you my estimate in a moment.

To get the capacitance of the linesman we have only to decide what radius of sphere best represents him. You may have a somewhat different estimate for the radius of a spherical linesman, but the capacitance should come out to be of the order of picoFarads.

If we say $a$ ~ 0.25 m (say), $C = 4\pi \times (8.85\times10^{-12}\ \text{Fm}^{-1})\times(0.25\ \text{m}) = 29$ pF

## Does it matter that the linesman is not a metal sphere?

We usually have a mental image of capacitors with metal plates – but any conductor, such as the human body, will have a capacitance, since charge will flow to keep it an equipotential. Of course, we saw in session 4 that capacitance is changed in the presence of a dielectric, which we're ignoring here.

So with alternating current, even without a complete circuit, as the linesman approaches the line, he will charge and discharge and at high enough potential a spark will cross the gap.

The charging and discharging will of course continue once the linesman is attached to the line. The cable can be live even if it is not a complete circuit. So even without a complete circuit, in the absence of a Faraday cage, there is a danger that a current will flow. Will this be sufficient to injure or kill the linesman? We must calculate the current to find out.

The amplitude of the AC current is the voltage divided by the impedance of the circuit. We'll do a rough estimate first and then a more detailed calculation.

For our rough estimate we neglect the resistance of the circuit. Then the alternating voltage induces an alternating charge on the capacitor given by Q=CV. The current is the time derivative so I = CdV/dt. But V is changing sinusiodally with frequency ω, so $V = V_0 \cos \omega t$ and $I = \omega V_0 \sin \omega t$. The amplitude of the current is therefore $I_0 = C\omega V_0$ .

For a voltage of 40 000 V at 50 Hz this gives a current of 4mA. Before we come to a conclusion let's check the validity of our estimate.

In more detail:

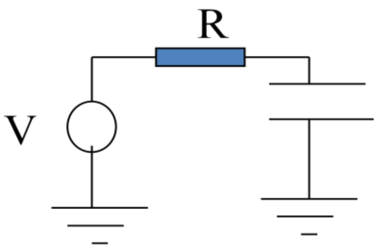
More accurately the pd across the circuit is

$$V = RI + Q/C = RdQ/dt + Q/C \qquad (1)$$

If $V = V_0 \cos\omega t$ then $Q = (V_0/\omega Z) \cos(\omega t + \phi)$, and $I = dQ/dt = -(V_0/Z) \sin(\omega t + \phi)$

We substitute back into equation (1) to find Z using the standard trigonometric identities for the sines and cosines of the sum of two angles. The phase phi is of no interest here, since it's not the power absorbed that kills. Equating coefficients of cosωt and sinωt gives $Z = (R^2 + 1/C^2\omega^2)^{1/2}$. So we compare 1/Cω to R: this is the same as the ratio of the time constant of the circuit, RC, to the period of the applied voltage, 1/50Hz or .02 seconds. So as long as R is sufficiently small, such that the time constant is less than the period, the current in the circuit will follow the applied voltage. For R< $10^9$ ohms this will be the case. Thus we can ignore the resistance in the circuit and our original estimate of the current will be good enough.

A current of 1mA with a direct pathway to the heart can be fatal, although it is around the threshold of perception for currents entering the hand. So we now understand where the danger lies.




## SAQs

1. The energy of a parallel plate capacitor with charge $Q$, area $A$ and plate separation $x$ is ½ $Q^2x/\varepsilon_0 A$. If the charge is held constant and the plates moved apart, what is the force.
   a) 0
   b) $Q^2/2\varepsilon_0 A$
   c) $Q^2/(4\pi\varepsilon_0 x^2)$
2. The same as question 1 except that the potential is held constant
   a) - $\varepsilon_0 A\ V^2/2x^2$
   b) $Q^2/2\varepsilon_0 A$
   c) $Q^2/(4\pi\varepsilon_0\, x^2)$
3. Does the charge on an isolated sphere tend to crush it or expand it?
   a) expand
   b) crush
4. The potential difference between two concentric cylindrical shells with radii $a$ and $b$ and surface charge density σ is $\sigma/\varepsilon_0 \log(b/a)$. What is the capacitance per unit length?
   a) $1/\varepsilon_0 \log(b/a)$
   b) $\varepsilon_0 \log(b/a)$
   c) $1/\varepsilon_0 \log(a/b)$

**The answers appear on the following page**

# Answers

1. a) Wrong: The energy is a function of the spacing, so it changes with the spacing; this must come from work done, which implies a force.

   b) Correct: Either from the expression for the force = dU/dx (where U is the potential energy), or directly because Work done = δ(energy) = δ($Q^2$x/A) = ($Q^2$/A)δx = Force x distance.

   c) Wrong: you're thinking of the force between two charges, but here there are two sheets of charges acting on each other, all at different distances apart. You could integrate over the plates to find the force, but as usual it's easier to consider a change in energy.

2. a)Correct: we have to express the energy from question 1 in terms of the fixed potential V, instead of the fixed charge Q, and . The difference in sign means that work is extracted – this is because there is a current flow supplied by the external circuit to keep the potential fixed.

   b) Wrong: The force is not the same if the potential is held fixed because charges have to flow to achieve this. The correct result is obtained by looking at the energy, 1/2CV $^{2,}$ as a function of x and other fixed quantities and seeing how it changes with x.

   c) This is the force between two point charges; but here the charge is distributed over the plates.

3. a) Correeect: we need the energy as a function of the fixed charge on an isolated sphere. This is ½$Q^2$/ 4π$\varepsilon_0$*r; so dU/dr* = - ½*Q*$^2$/ 4π$\varepsilon_0$*r*$^2$ which means that the force is positive outwards, tending to increase r. If you got this answer by thinking that like charges repel you're right, but you won't be able to solve more complex problems on this basis.

   b) Wrong: you can't reason this out in words – you have to calculate the change in energy for a fixed charge as the radius of the sphere is increased.

4. a) Wrong: you've forgotten to invert it: V=σ/C (where C is per unit length)

   b) Correct: V=σ/C (where C is per unit length)

   c) Wrong: this is not how you invert logs!

# Currents and Magnetic Fields

Here we'll return to the safety of the linesman with a problem that involves magnetic fields. This will be our first introduction to magnetism, which is quite a difficult but interesting and important subject. We'll be taking it up again in the next sessions.

There are two extreme views on the cancer causing potential of magnetic fields. These are represented by this and the next page. The purpose of this discussion is not to add to that debate, but to calculate the strengths of the magnetic fields involved. So we'll leave you to read the debate on this and the next page if you're interested.

**Power Lines and Cancer: Nothing to Fear**
**John W. Farley, Ph.D.**

*The notion that electric power lines can cause cancer arose in 1979 with a single flawed epidemiological study that created a stir. Subsequent epidemiologic and animal studies have failed to find a consistent and significant effect. No plausible mechanism linking power lines and cancer has been found. In recent years, the verdict from large-scale scientific studies has been conclusively negative, and scientific and medical societies have issued official statements that power lines are not a significant health risk. In short, there is nothing to worry about.*

*Childhood leukemia can be used as an indicator that radiation exposure is sufficient to cause illness, because radioactivity elevates rates of leukemia before it produce other forms of cancer. Consequently, childhood leukemia ought to be the easiest to detect. In 1979, two researchers, Nancy Wertheimer and Ed Leeper, published an article based on their own epidemiologic study, alleging that the incidence of childhood leukemia was higher in Denver neighborhoods that were near electric power lines [1]. Their article generated a flurry of other studies. The idea was picked up by Paul Brodeur, who wrote a frightening three-part article for The New Yorker that reached a large and influential audience. Subsequent books by Brodeur in 1989 and 1993 alleged that power lines were "Currents of Death" and that the power industry and the government were engaged in a cover-up [2,3]. The journal Microwave News has consistently echoed Brodeur's message.*

….

*By the mid-90s, at least 100 epidemiologic studies had been published. Most found no correlation between cancer and measured powerline magnetic fields in houses. The evidence accumulated that power lines are not a health risk. ….  One prominent panel, assembled by the Oak Ridge Associated Universities, concluded:*

*There is no convincing evidence in the published literature to support the contention that exposure to extremely low-frequency electric and magnetic fields generated by sources such as household appliances, video display terminals, and local power lines are demonstrable health hazards." [5]*

http://www.quackwatch.org/01QuackeryRelatedTopics/emf.html

From ***Midwest Today*, April/May 1996** :

*A growing body of scientific evidence suggests that invisible electromagnetic fields (EMFs) -- created by everything from high-voltage utility company lines to personal computers, microwave ovens, TVs and even electric blankets -- are linked to a frightening array of cancers and other serious health problems in children and adults.*

*Though it received scant attention from the mainstream press, a report leaked last October from the U.S. National Council on Radiation Protection said there is a powerful body of impressive evidence showing that even very low exposure to electromagnetic radiation has long-term effects on health.*

*The report cited studies that show EMFs can disturb the production of the hormone melatonin, which is linked with sleep patterns. It said there was strong evidence that children exposed to EMFs had a higher risk of leukemia.*

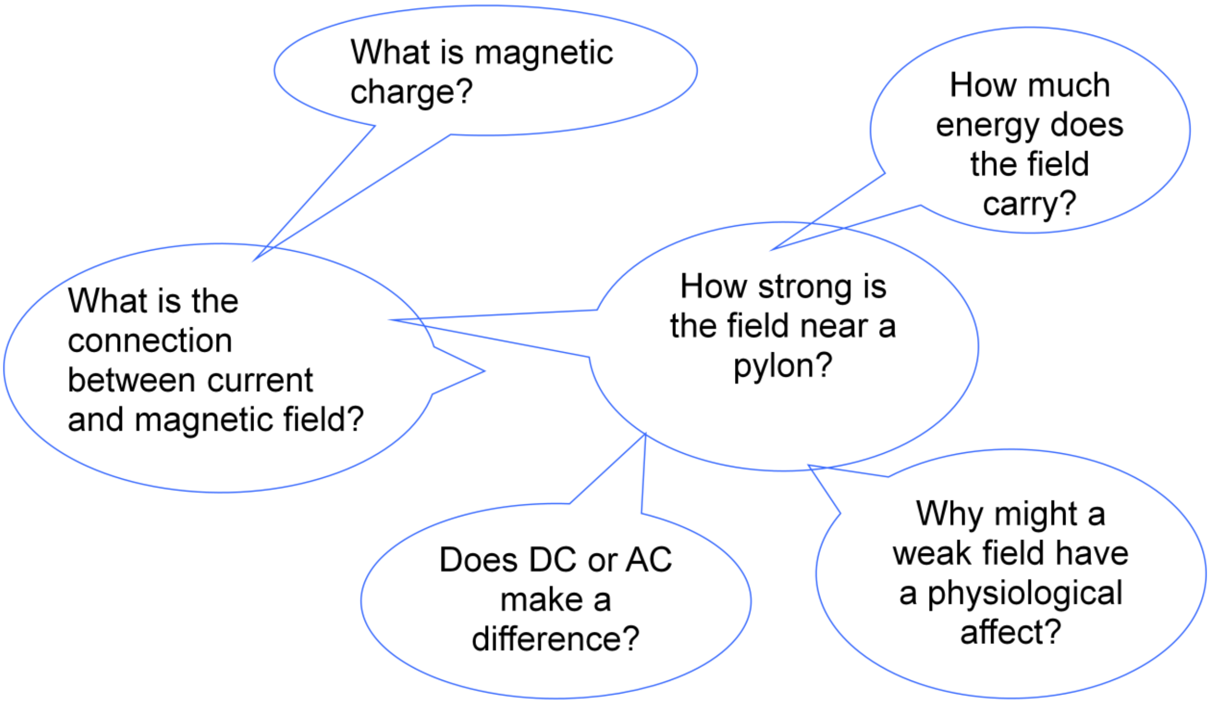
*….*

*Now a surprising new report released in February by physicists at Britain's University of Bristol shows that power lines attract particles of radon -- a colorless, odorless gas irrefutably linked with cancer.*

As I said, we're not going to discuss which is right? But what can we say about the strength of a magnetic field near a power line?

**Draw a concept map of the questions you might ask here before looking at ours.**

Clearly we should begin by asking what generates the magnetic field. Presumably we need to relate the current in the power line to the field that it produces. We'll look at more fundamental questions on magnetism later. Think about the questions you might ask before looking at our list.



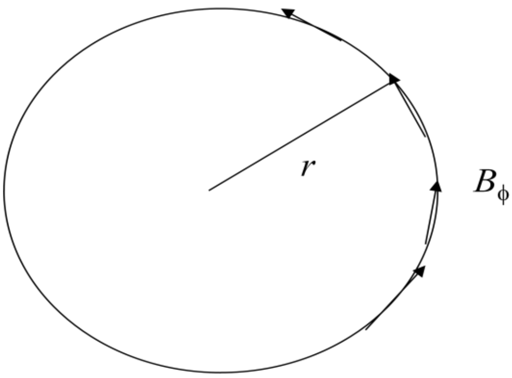

## Circulation

Magnetic fields come in two guises: form magnets and currents. In fact these are related – magnets arise from loops of current - but the connection isn't straightforward. So we'll start with magnetic fields from currents and come back to magnetic materials.

First we need the concept of circulation. Think of cars going round a roundabout. The same circulation of traffic can arise from well spaced out cars on a big roundabout, or congested ones on a small roundabout. It also depends on the speed of the cars. In other words it's a measure of the intensity of traffic flow round a closed curve.

Image[4]

The circulation of any field, such as the magnetic field *B,* is measured in a similar way: it's the magnitude of the tangential component of the field $B_\phi$, times the circumference of the curve around which it flows. If the field is not uniform round the curve, then the circulation is obtained from an integral round the curve.




We use the concept of circulation to relate a current to the magnetic field it generates.

[4] Ondulations, T, as obtained from www.flickr.com, Creative Commons Licensed

# How currents create magnetic fields

Ampere's law:

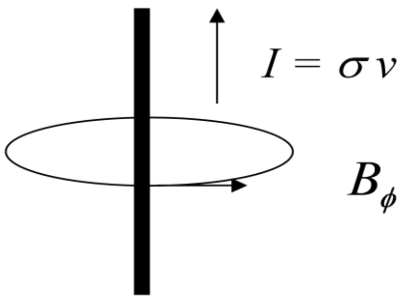
> *Circulation of the magnetic field is equal to the encircled current (up to a factor that depends on the units)*

In the SI system current is measured in amps, the magnetic field in tesla and the constant is $\mu_0 = 4\pi \times 10^{-7}$ henries per metre. Note that $B_{phi}$ is constant round the circle. Can we apply this to AC?

The circulation round a circle is $2\pi r B_\phi = \mu_0 I$. Can we derive the magnetic analogue of Coulomb's law?

# Relation between electricity and Magnetism

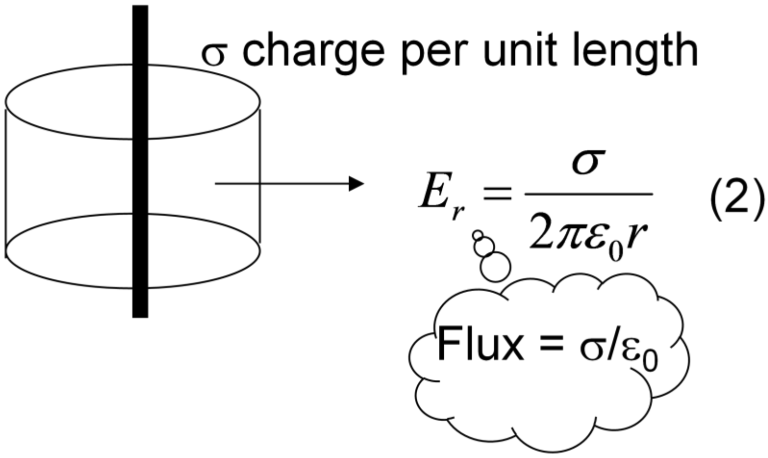
There are various ways of exploring the relation between electricity and magnetism. Here's one of them, which will be useful when we come to try to understand the power line problem. Imagine a current made up of a charge per unit length σ flowing with speed v. The current (the charge per second passing a fixed point) is then σv. Writing the magnetic field in terms of this current we get equation (1).




$$B_\phi = \frac{\mu_0}{2\pi}\frac{I}{r} = \frac{\mu_0}{2\pi}\frac{\sigma v}{r}$$

(1)

Now compare this with the electric field of the charge in the lower panel. Take a unit length of charge and apply Gauss's theorem to a cylinder surrounding it. The flux through the cylinder is $2\pi r E_r$ and this must equal the enclosed charge over $\varepsilon_0$. This gives us the expression for the electric field $E_r$ in (2).




Equations (1) and (2) look quite similar in form: the relation between them is equation (3). The product $\varepsilon_0\mu_0$ turns out to be $1/c^2$, where c is the speed of light, so we can write the relation between the magnetic and electric fields of the moving charge as (4).

Relation between $E$ and $B$:

$$B_\phi = \mu_0\varepsilon_0 v E_r \qquad (3)$$

$$= \frac{vE_r}{c^2} \qquad (4)$$

Of course, the electric and magnetic fields have different directions: we can include that by writing the relation in terms of the vector product as in (5), for those of you who are comfortable with this notation.

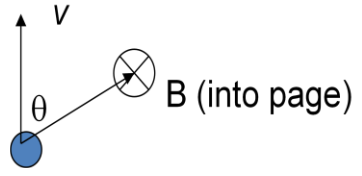
In vector form

$$\boldsymbol{B} = \frac{\boldsymbol{v}\times\boldsymbol{E}}{c^2} \qquad (5)$$

The surprising thing is that this is not just a relation that holds for infinite straight wires, but a quite general relation – it's known as the Biot-Savart law, although this is not the conventional way the law is expressed. For the particular case of a point charge moving with speed v the law takes the form (6) – by substituting Coulomb's law for the electric field of the charge. The various directions are given in the figure. So this is the magnetic field of a point charge. Notice three things: the magnetic field of the charge is zero if the charge is not in motion; and secondly the factor of $c^2$ in the denominator. Finally, the magnetic effects of moving charges are much less than their electric effects, in this case by a factor of $1/c^2$.

For a point charge in motion:

$$B = \frac{qv\sin\theta}{4\pi r^2 c^2} \qquad (6)$$




Let's apply this to the field of the high tension cable. The fact that the current is alternating means that so is the field.

The fact that the current is alternating means that so is the field.

500 MW cable at 500kV
The field is 66 mG (= 66 μTesla) at a distance of 30 m
For a single cable this is equivalent to 1000 A

We get a magnetic field of the order of 66 micro Tesla. It is interesting to compare this with various fields we all encounter in everyday life, in particular static fields such as the Earth's magnetic field, and varying ones such as those associated with electronic equipment.

In fact, power lines are arranged to minimise fields by having multiple wires with opposed current flows.

There is another component to the field which arises from the varying E-field. In a future session we shall see that a varying E field gives rise to a B field. This is in addition to the magnetic field we have calculated here and is called a radiation field.

**What is the field near a mobile phone, TV set,…**

See: http://www.calpoly.edu/~dhafemei/background2.html

http://www.who.int/peh-emf/about/WhatisEMF/en/index3.html

# Summary of the linesman problem

If the line were DC the potential difference between the line and the linesman would generate high electric fields ($E = dV/dx$) between sharply curved objects. This would lead to the disruption of air molecules causing the air to become conducting and sparks to jump the gap.

With an AC the capacitance of the linesman to Earth provides a circuit and hence strong electric fields which produce sparking.

The sparks are caused by currents which are flows of charge.

Conductors are equipotential surfaces. Within a closed conductor the electric field is zero. This can be obtained from Gauss's law which states that the flux of the electric field equals the enclosed charge (up to a constant that depends on the units).

The current in the circuit can be calculated from the capacitance of the linesman.

Ampere's law states that the circulation of a magnetic field equals the enclosed current (up to a constant that depends on units). This can be used to calculate the magnetic fields experienced by the linesman.

# Additional problems:

## 1. How does the Sun shine

Protons in the Sun have to come close enough together in order to produce the fusion power that keeps the Sun shining. They are however held apart by their Coulomb repulsion. In this problem we calculate how close two protons of given energy will approach before the Coulomb force turns them back. If we think of the initial state as the energetic protons far apart and the final state as the point at which they come to rest, this is a before and after question that can be solved by conservation of energy.

The potential energy is positive relative to the starting point because energy must be extracted to restore the proton to its original distance. The total energy is positive because this is not a bound system.

How close can two protons approach at a given energy?
Total Energy = KE + PE = constant
Initially PE = 0, KE = 2E (say)
Finally KE = 0, PE = $e^2/4\pi\varepsilon_0 r$
Therefore $r = 8\pi\varepsilon_0 E/e^2$

**Why is the potential energy positive?**

The distance we obtain is much larger than that required for a nuclear reaction by about a factor of 10. There are two ways in which this is resolved. First only the most energetic particles take part in the reaction, not the average ones. That's why the Sun is not a bomb. Second quantum mechanics comes into play and allows particles to interact at larger separations than classically.

If we take E = the mean energy of a proton at $2\text{x}10^7$K = $4.4 \times 10^{-16}$ J then $r = 1.4 \times 10^{-13}$ m. This is too large for nuclear reactions.

## 2. Lightning

The Franklin experiment is as follows:

*While waiting for completion of the spire, he got the idea of using a flying object, such as a kite, instead. During the next thunderstorm, which was in June 1752, he raised a kite, accompanied by his son as an assistant. On his end of the string he attached a key and tied it to a post with a silk thread. As time passed, Franklin noticed the loose fibers on the string stretching out; he then brought his hand close to the key and a spark jumped the gap. The rain which had fallen during the storm had soaked the line and made it conductive.*

*From Wikipedia*

A lightning strike carries a current of about 30 000 A. What is the magnetic field generated by lightning? How does this compare with the field from power lines?

# Meta tags

Author: Naomi Banks.

Owner: University of Leicester

Title: Enhancing Physics Knowledge for Teaching – Electric and Magnetic Fields

Keywords: Magnetic fields; electrical potential; capacitance; sfsoer; ukoer

Description: This session will introduce you to electric and magnetic fields. We'll look at what we mean by a field and at what electric charge is and relate the two through Gauss's theorem. From this we'll derive Coulomb's law for the force between charges. Then we'll look at the concept of electrical potential which is related to the work done in moving a charge through a field. We'll return to the notion of capacitance which we used in session 4, this time looking at how capacitance is computed. Finally we'll take out first look at magnetism from the point of view of the magnetic effects of current.



Language: English

Version: 1.0

# Additional Information

This pack is the Version 1.0 release of the module. Additional information can be obtained by contacting the Centre for Interdisciplinary Science at the University of Leicester.
http://www.le.ac.uk/iscience